LONDON METAL EXCHANGE

REVIEW OF METHODS FOR THE INTERPOLATION OF CONTRIBUTED WING VOLATILITIES

Introduction

1 At present, option valuation for daily variation margining by LCH.Clearnet is performed by using the at-the-money ("ATM") volatility for all strikes. It is recognised by the LME and market users that providing one ATM volatility is not sufficient to provide a sound basis for marking options positions to market, whether it be for clients' valuations or for LCHC margining purposes. It is for this reason that a new system is being implemented by the LME to collect closing volatilities at 5 delta points (-10, -25, 50, 25, 10) for each option series from contributing members that are active in LME Options markets. LME intends to publish the data to provide better price transparency to reflect the volatility skews that may be present in each of these series.

Findings

Contributions by 50 delta

2 The LME has considered two options for the definition of the ATM point.

(a) ATM = the forward

(b) ATM = strike at which the delta of the put = the delta of the call =50 delta

3 The LME has chosen the second option as it gives better spacing for longer dated contracts and will provide a more useful surface point for interpolation.

4 The Nickel 123 month ATM strike can be seen to be quite close to the 25 delta interpolated strike, whereas the 50 delta strike is much closer to the centre. This could have consequences for interpolation as the data would be less well spaced out with ATM. If the ATM-forward strike came too close to the 25 delta strike, this would result in there only being four real contributions instead of five. An example below of Nickel 123 month volatilities highlights this point.

	25d	ATMfwd	50d	25d
Ni 123m	17,783	19,429	40,319	92,410

5 Parallel to this initiative, there have been many debates over the best methodology to be used to interpolate the 5 point volatility surface into recognised strikes and how that data will be used and published. The LME has undertaken a study of the different methodologies of interpolation and has concluded that it should not be performing any interpolation themselves but will publish the volatilities in the original format as they are collected for -/+10, -/+25 and 50 deltas.

LME criteria in selection of methodologies

6 The LME analysed and reviewed four different interpolation methodologies available and their impact. Two criteria were used in selecting the interpolation methods in the analysis. First is simplicity of implementation and second is robustness in operation. Complex methods that required different types of fit, depending on the data were excluded. The selected methods had to work with only five data points (-10, -25, 50, 25, 10 deltas) which meant that regression models and

various smoothing algorithms, such as Hagan's SABR model, Kernel methods and all the stochastic and local volatility models were ruled out.

Scope of analysis

7 Four metals were examined: Copper, Nickel, Zinc and Aluminium. Volatilities for 13 prompts per metal from 1 week, 1 month, 3 months, 6 months, 9 months, 12 months, 15 months, 21 months, 27 months, 39 months, 51 months, 63 months and 123 months were used, where available. Volatilities were given for ATM strike, in conformance with current practice, -/+10 and -/+25 delta, and also for 50 delta for comparisons. The methods were all within the framework of a five-point surface in two dimensions i.e. separate fit for each series.

Objectives

8 The objectives of the analysis were:-

(a) To compare each of the fits using the selected methods to a base case scenario (cubic spline is chosen, which requires no parameter fitting[1]).

(b) Highlight the maximum differences between each method and the base case.

(c) To compare the impact of a shift from one surface to the next by looking at percentage changes to option deltas and to option premia.

Methods used

9 The methods selected for the analysis were: -

(a) Linear interpolation in strike space.

(b) Linear interpolation in delta space.

(c) The SVI parametric model[2]. This is straightforward, readily adaptable to our data and it is designed to work in strike space[3].

(d) Cubic spline.

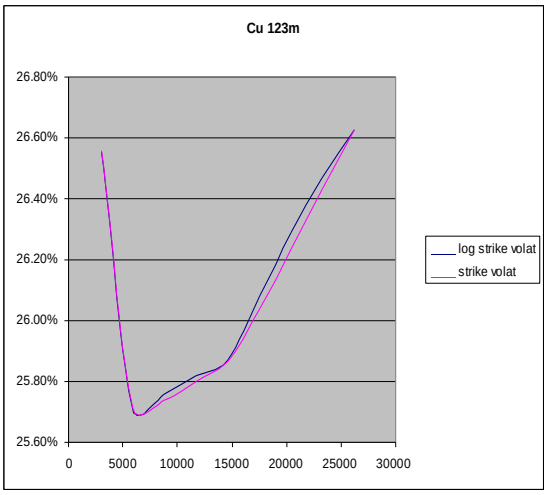
Linear Interpolation in Strike Space

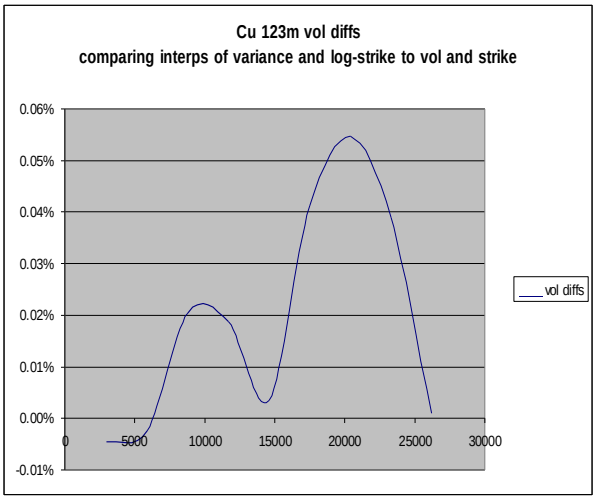
10 The contributed volatilities are at five points: ATM-forward, and for out-of-the-money puts and calls respectively: -10, -25, 25 and 10, which are averaged at each delta. If we imply the strikes for each delta using the relevant part of the Black 76 Option Pricing Formula to obtain a span of strikes from in-the-money to out-of-the-

[1] Tests were made with the cubic spline methodology across the data to ensure that it was possible to obtain realistic interpolations. We will show that this is not always possible with cubic splines.
[2] The parametric model is the SVI ("stochastic volatility inspired") model published by Professor Jim Gatheral in 2004 and later in "The Volatility Surface", Wiley, 2006.
[3] A caveat for the SVI model is that the daily re-estimation of its parameters may throw up problems in an operational environment.

money, linear interpolation for the intermediate strikes can then be carried out. This is called linear interpolation in "strike space". This method does not solve the problem of extrapolating to far strikes.

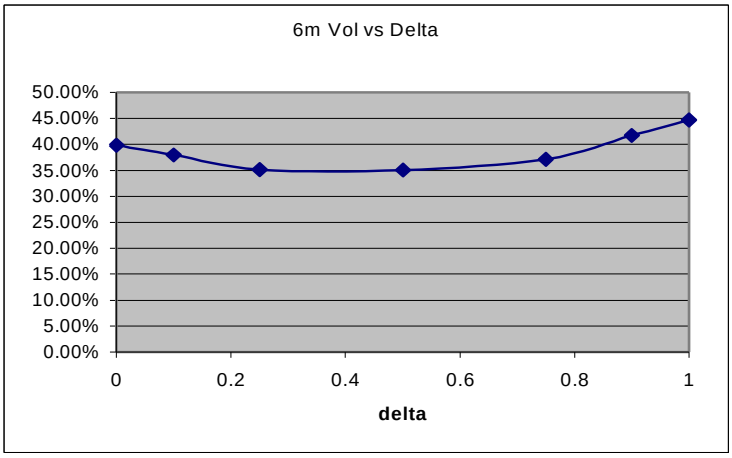
11 Another interesting alternative method that has been advocated is to use starting assumptions similar to SVI and to interpolate variances in "log-strike" space ie interpolating variance from the log-strikes log(F/K) and to back out the volatilities against strike. We are already performing interpolation in delta space and SVI in this study, and so chose to examine simple linear interpolation in strike space rather than this alternative.





Linear Interpolation in Delta Space

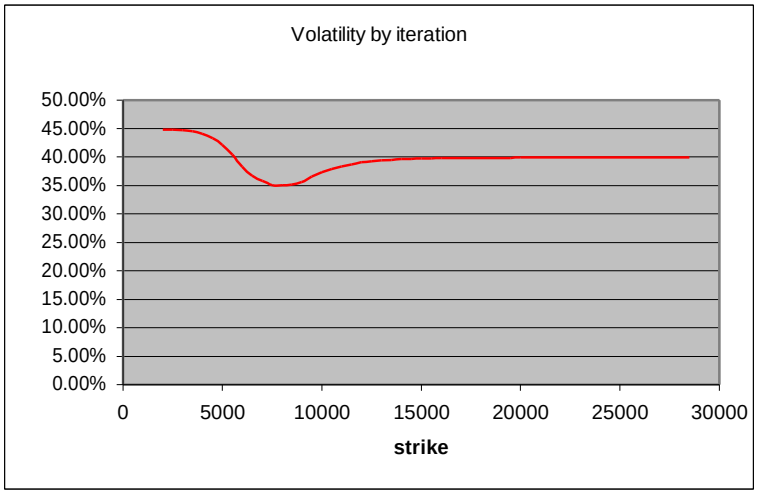
12 One way to deal with far strikes is to carry out linear interpolation in "delta space". The deltas have first to be re-expressed as all for calls ie 90, 75, 50, 25 and 10. Then it is easy to extrapolate the volatility for 100 delta and zero delta without obtaining unrealistically large volatilities. This sets a cap on the volatilities for strikes that are far away from the money. For example we take 10/15ths of the difference in volatilities between 10 and 25 delta and add it to the10 delta volatility to calculate the volatility at zero delta.



13 It is straightforward to perform linear interpolation of volatility for a corresponding delta value in the graph above. To find volatility for a specified strike is less straightforward when in delta space. This is because if we apply the formula

for implied strike to the value we have for an interpolated volatility by delta, the resulting strike follows from the calculation and it is not known in advance what its value will be. We can repeat this calculation until we obtain a particular strike we want.

14 This process is systematised so that we iteratively home in on the volatility for a given strike. This is done by specifying the strike for which we require a volatility, and making a guess as to its volatility. We calculate the corresponding delta (using Black76) as an intermediate step. This delta-volatility pair will not lie on the curve eg in the above graph. By interpolating the volatility from the graph that corresponds to our calculated delta, we can compare the two volatility values: the interpolated value and our guess. They would be the same for the correct guess of volatility for the specified strike. As they are unlikely to be, we can subtract them and solve iteratively for a zero difference.



15 This gives a consistent set of volatilities for all the strikes we need, which are capped by the zero delta and 100 delta values.

The SVI ("stochastic volatility inspired") Model

16 The SVI Model of Jim Gatheral is $\sigma_{BS}^2(k) = a + b\left\{\rho(k-m) + \sqrt{(k-m)^2 + \sigma^2}\right\}$ where a, b, ρ, m and σ are parameters that govern the positioning of the curve about the contributed data. The variable k = log(K/F) where F is the closing futures price and K is the strike price. The variable $\sigma_{BS}^2(k)$ is the variance calculated corresponding to the desired strike price ie it is a function of k and the five parameters. The parameters have to be fitted for each set of contributed volatilities (ie daily if this were done for real), in our case prompt by prompt. This is done so that the modelled values of variance fit as closely as possible to the contributed variances (ie volatilities squared), by varying the parameters until this is achieved. Operating a Solver function, in this study, EXCEL's© Solver, is used to achieve this.

Cubic Spline Interpolation

17 Cubic spline interpolation estimates a smooth curve that goes through all the points and which is differentiable at the points ie there are no sharp discontinuities.

This can constrain the curve when it has to make steep turns to fit the data, causing over- and under-shooting. It was verified that this did not cause a problem with the main body of data used for comparison of the four methods.

18 Because cubic spline does not require parameter estimation, it is an attractive method for testing the different methods, as the problem with fitting to some data can be checked beforehand.

Results

19 ***Appendix 1*** shows the summary of worst differences to Base Case. Comparisons are made between cubic spline volatilities and the other three methods to see the impact of moving from one surface to another. The worst and average impacts are tabulated below, for actual differences in volatility, and percentage differences in deltas and in call premiums.

20 ***Appendix 2A*** shows the full tables of impacts, for each metal across all 13 prompts, from which Appendix 1 was derived. The tables are organised by difference to volatility for each metal, then percentage difference to delta, then premiums.

21 ***Appendix 2B*** shows % delta differences.

22 ***Appendix 2C*** shows the % call premium differences.

23 ***Appendix 3*** contains the volatilities from the four interpolation methods employed.

24 ***Appendix 4*** shows results using 50 delta across the linear and non-linear method of interpolation.

25 **Linear in Strike space:** This is an approximation to linear in delta using the inputs by delta and the assumption that volatility is linear with delta. Prices are likely to be arbitrageable.

26 **Linear in Delta space:** This produced the most different results in the higher strikes, before coming back into line with the other methods. However it was not so different as to be in a class of its own, regarding the option premia and impact on deltas. Prices are likely to be arbitrageable.

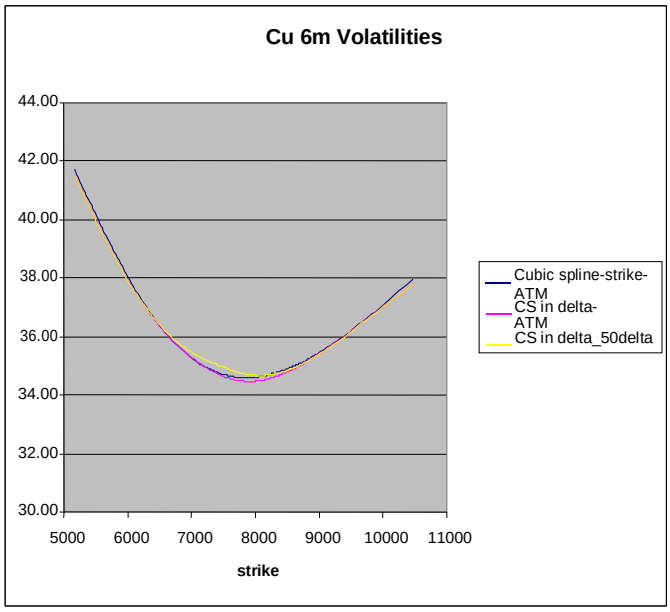
27 **SVI:** The methodology is simple and produced a smooth curve at the money. However the difficulty that we found in establishing the starting parameters for the SVI model, means that judgement has to be applied for use in an operational environment. While parameters can be established by making more than one attempt, this is not robust and stable enough for daily production.

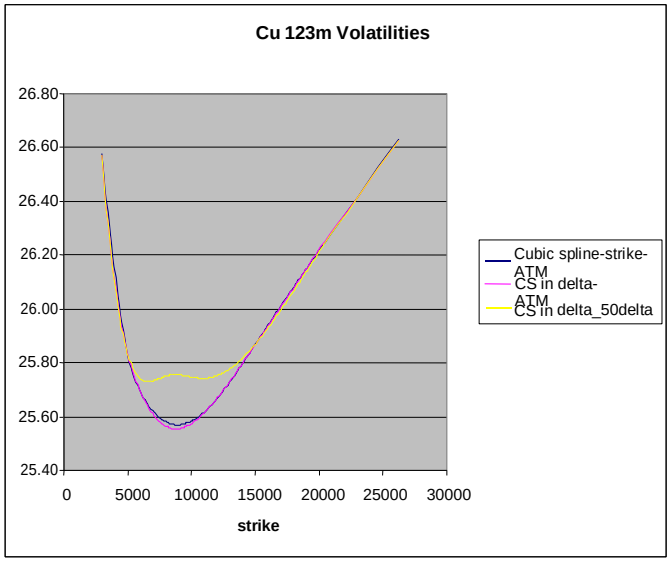
28 Further, in the examples, a change to one value can affect the whole curve even though nothing else has changed. This could be problematic for variation margins where a member will have expectations about the variation margin.

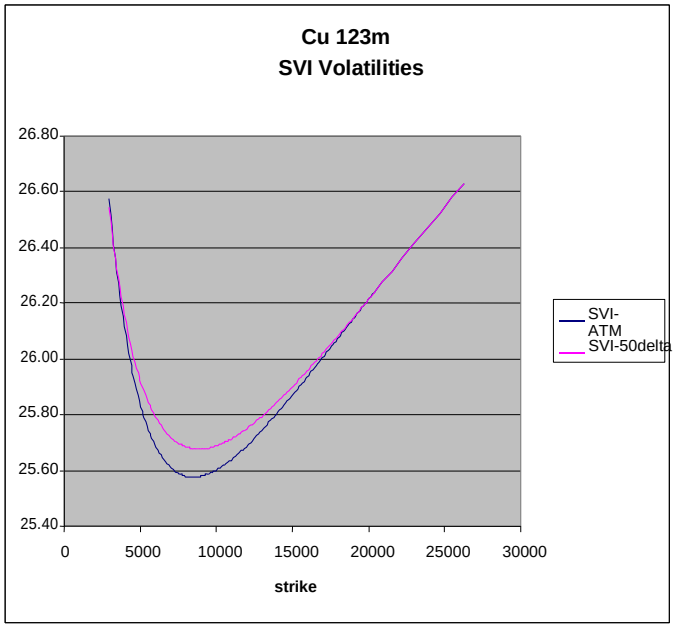
Cubic Spline Test

29 We found that the model could overfit occasionally, although this was not a problem in the study, except for the case of some months with 50 delta volatilities (which was not the basis for comparison). Further, cubic spline interpolation can be affected sometimes by a change to one value affecting the whole curve. This again can be problematic. Overall, the results from SVI and cubic spline were very similar. The results of tests carried out for cubic splines are shown below, to demonstrate the kink that can occur when attempting to fit through the 5 points. In this case, the ATM volatility was replaced with the 50 delta volatility, causing the problem shown in the Copper 123 month contract.

30 The corresponding SVI result is also shown, where the flexibility of SVI, in not having to go through the points, allows for a better overall fit.







31 These graphs showing the kink when using cubic splines illustrate where arbitrage opportunities could be present as a direct result of the interpolation method, even though the input 50 delta volatility was a little lower than the surrounding 25 delta volatilities.

32 In fact, given appropriate contributed volatilities, all of the interpolation methods we examined, with the notable exception of SVI, could allow arbitrage opportunities. By allowing for a flexible fit, SVI produces a smooth curve even if the contributed volatilities would produce kinks under the other interpolation methods.

33 This shows the possible instability of the non-linear methods, as alteration to one value can shift a segment of the curve or even the whole curve. This may have undesirable consequences for some variation margins, if the effects were noticeable for the portfolio.

Conclusions

34 The LME recognise that there are many other hybrid ways of interpolation.

35 LCH.Clearnet (LCHC) is the Clearer for LME options contracts and expressed a preference for a simple and robust methodology to apply to their daily margining system. Linear interpolation in strike space may be used by LCH.Clearnet for generation of volatilities used in the calculation of all variation and initial margins. Clearly this is an improvement to the current flat volatility method which can be hugely inaccurate.

36 The LME has to decide how it will publish the wing volatilities. It is proposed that LME should collect 50 delta instead of ATM forward from contributing members and publish this to the market together with -/+10 and -/+25 delta on either side. It is also proposed that LME should publish to the market the five volatility data for each series in the format of -10, -25, 50, 25, and 10 in delta space without converting them into recognised strike gradations. It is up to LCHC, LME members and market users to choose their preferred methodology for interpolation between and beyond these five points.

Appendix 1 - Summary of Worst Differences to Base Case

	Max	Min	Median	Mean
Cu: Worst Vol Diffs all Prompts				
cs-SVI	0.13	-0.04	0.01	0.03
cs-lin in strike	0.01	-0.50	-0.17	-0.19
cs-lin in delta	0.72	-0.51	-0.26	-0.25
	Max	Min	Median	Mean
Ni: Worst Vol Diffs all Prompts				
cs-SVI	0.09	-0.04	0.01	0.01
cs-lin in strike	0.07	-0.23	-0.09	-0.10
cs-lin in delta	0.40	-0.28	-0.14	-0.14
	Max	Min	Median	Mean
AL: Worst Vol Diffs all Prompts				
cs-SVI	0.06	-0.05	0.01	0.01
cs-lin in strike	0.07	-0.10	-0.03	-0.03
cs-lin in delta	0.22	-0.22	-0.08	-0.10
	Max	Min	Median	Mean
Zn: Worst Vol Diffs all Prompts				
cs-SVI	0.07	-0.03	-0.01	0.01
cs-lin in strike	0.03	-0.15	-0.05	-0.06
cs-lin in delta	0.20	-0.21	-0.10	-0.10

	Max	Min	Median	Mean
Cu: Worst %Delta Diffs all Prompts				
cs-SVI	1.92%	-2.78%	-0.49%	-0.46%
cs-lin in strike	0.11%	-1.36%	-0.14%	-0.36%
cs-lin in delta	0.24%	-0.24%	-0.01%	0.03%
	Max	Min	Median	Mean
Ni: Worst %Delta Diffs all Prompts				
cs-SVI	1.32%	-1.32%	-0.44%	-0.48%
cs-lin in strike	0.66%	-0.38%	-0.10%	-0.11%
cs-lin in delta	0.21%	-0.24%	0.06%	0.06%
	Max	Min	Median	Mean
AL: Worst %Delta Diffs all Prompts				
cs-SVI	1.26%	-1.31%	-0.35%	-0.40%
cs-lin in strike	0.90%	-0.16%	-0.02%	0.25%
cs-lin in delta	0.22%	-0.18%	-0.02%	-0.03%
	Max	Min	Median	Mean
Zn: Worst %Delta Diffs all Prompts				
cs-SVI	0.73%	-0.95%	-0.48%	-0.44%
cs-lin in strike	0.26%	-0.42%	-0.08%	-0.10%
cs-lin in delta	0.11%	-0.23%	-0.01%	-0.04%

	Max	Min	Median	Mean
Cu: Worst Call Premium %Diffs all Prompts				
Call prem cs-lin in delta	3.39%	-4.47%	-0.90%	-1.06%
Call prem cs-lin in strike	0.15%	-2.92%	-0.48%	-0.87%
Call prem cs-svi	0.48%	-0.38%	0.04%	0.07%
Call prem cs-flat	33.57%	-2.63%	2.19%	3.24%
	Max	Min	Median	Mean
Ni: Worst Call Premium %Diffs all Prompts				
Call prem cs-lin in delta	2.36%	-2.01%	-0.87%	-0.83%
Call prem cs-lin in strike	0.89%	-0.81%	-0.23%	-0.28%
Call prem cs-svi	0.39%	-0.34%	0.11%	0.09%
Call prem cs-flat	27.78%	-0.05%	3.98%	7.68%
	Max	Min	Median	Mean
AL: Worst Call Premium %Diffs all Prompts				
Call prem cs-lin in delta	2.43%	-2.20%	-0.78%	-0.81%
Call prem cs-lin in strike	1.34%	-0.51%	0.21%	0.39%
Call prem cs-svi	0.36%	-0.40%	-0.05%	-0.05%
Call prem cs-flat	55.78%	-1.68%	12.29%	17.24%
	Max	Min	Median	Mean
Zn: Worst Call Premium %Diffs all Prompts				
Call prem cs-lin in delta	1.20%	-1.46%	-0.81%	-0.72%
Call prem cs-lin in strike	0.38%	-0.78%	-0.16%	-0.21%
Call prem cs-svi	0.27%	-0.35%	-0.02%	-0.04%
	21.55%	-0.02%	4.90%	1.22%

Appendix 2A - Full Tables of Differences to Base Case

Volatility Differences

Cu	Differences in Volatility	Max	Min	Median	Mean
1w	cs-SVI	0.07	-0.02	0.01	0.02
1w	cs-lin in strike	0.00	-0.23	-0.10	-0.11
1w	cs-lin in delta	0.72	-0.23	0.16	0.12
1m	cs-SVI	0.04	-0.01	0.00	0.01
1m	cs-lin in strike	0.00	-0.32	-0.13	-0.14
1m	cs-lin in delta	0.44	-0.32	0.00	-0.03
3m	cs-SVI	0.06	-0.01	0.00	0.01
3m	cs-lin in strike	0.00	-0.40	-0.15	-0.16
3m	cs-lin in delta	0.24	-0.41	-0.17	-0.15
6m	cs-SVI	0.08	-0.02	0.01	0.01
6m	cs-lin in strike	0.00	-0.46	-0.17	-0.19
6m	cs-lin in delta	0.13	-0.47	-0.25	-0.23
9m	cs-SVI	0.10	-0.03	0.01	0.01
9m	cs-lin in strike	0.00	-0.48	-0.17	-0.19
9m	cs-lin in delta	0.08	-0.48	-0.26	-0.25
12m	cs-SVI	0.11	-0.02	0.01	0.02
12m	cs-lin in strike	0.00	-0.48	-0.16	-0.18
12m	cs-lin in delta	0.03	-0.49	-0.24	-0.24
15m	cs-SVI	0.11	-0.03	0.01	0.02
15m	cs-lin in strike	0.00	-0.48	-0.15	-0.17
15m	cs-lin in delta	0.03	-0.49	-0.22	-0.24
21m	cs-SVI	0.12	-0.04	0.01	0.02
21m	cs-lin in strike	0.00	-0.50	-0.15	-0.18
21m	cs-lin in delta	0.02	-0.51	-0.22	-0.24
27m	cs-SVI	0.13	-0.04	0.01	0.03
27m	cs-lin in strike	0.00	-0.50	-0.14	-0.19
27m	cs-lin in delta	0.02	-0.51	-0.25	-0.25
39m	cs-SVI	0.13	-0.03	0.01	0.03
39m	cs-lin in strike	0.00	-0.49	-0.13	-0.19
39m	cs-lin in delta	0.03	-0.50	-0.21	-0.23
51m	cs-SVI	0.10	-0.02	0.01	0.02
51m	cs-lin in strike	0.00	-0.38	-0.11	-0.17
51m	cs-lin in delta	0.01	-0.40	-0.16	-0.19
63m	cs-SVI	0.07	-0.02	0.00	0.01
63m	cs-lin in strike	0.00	-0.27	-0.09	-0.13
63m	cs-lin in delta	0.01	-0.29	-0.11	-0.14
123m	cs-SVI	0.05	-0.02	0.00	0.00
123m	cs-lin in strike	0.01	-0.18	-0.04	-0.06
123m	cs-lin in delta	0.00	-0.21	-0.09	-0.09

Ni	Diff in volatility	Max	Min	Median	Mean
1w	cs-SVI	0.04	-0.02	0.01	0.01
1w	cs-lin in strike	0.00	-0.17	-0.08	-0.08
1w	cs-lin in delta	0.40	-0.14	0.09	0.09
1m	cs-SVI	0.06	-0.02	0.01	0.01
1m	cs-lin in strike	0.00	-0.22	-0.09	-0.10
1m	cs-lin in delta	0.30	-0.21	0.00	0.00
3m	cs-SVI	0.07	-0.02	0.00	0.01
3m	cs-lin in strike	0.00	-0.23	-0.07	-0.10
3m	cs-lin in delta	0.17	-0.26	-0.08	-0.08
6m	cs-SVI	0.07	-0.02	0.00	0.01
6m	cs-lin in strike	0.00	-0.21	-0.07	-0.09
6m	cs-lin in delta	0.11	-0.27	-0.12	-0.11
9m	cs-SVI	0.07	-0.02	0.00	0.01
9m	cs-lin in strike	0.00	-0.20	-0.06	-0.07
9m	cs-lin in delta	0.06	-0.27	-0.11	-0.12
12m	cs-SVI	0.05	-0.04	0.00	0.00
12m	cs-lin in strike	0.00	-0.18	-0.06	-0.07
12m	cs-lin in delta	0.03	-0.27	-0.12	-0.13
15m	cs-SVI	0.06	-0.02	0.00	0.00
15m	cs-lin in strike	0.00	-0.19	-0.04	-0.07
15m	cs-lin in delta	0.02	-0.27	-0.13	-0.14
21m	cs-SVI	0.06	-0.02	0.00	0.00
21m	cs-lin in strike	0.00	-0.19	-0.03	-0.06
21m	cs-lin in delta	0.01	-0.28	-0.14	-0.14
27m	cs-SVI	0.05	-0.02	0.00	0.00
27m	cs-lin in strike	0.00	-0.16	-0.02	-0.05
27m	cs-lin in delta	0.01	-0.27	-0.14	-0.13
39m	cs-SVI	0.04	-0.03	0.00	0.00
39m	cs-lin in strike	0.01	-0.08	-0.01	-0.02
39m	cs-lin in delta	0.01	-0.18	-0.09	-0.08
51m	cs-SVI	0.05	-0.03	-0.01	0.00
51m	cs-lin in strike	0.02	-0.07	-0.01	-0.01
51m	cs-lin in delta	0.01	-0.17	-0.09	-0.08
63m	cs-SVI	0.09	-0.03	0.00	0.01
63m	cs-lin in strike	0.03	-0.10	-0.01	-0.02
63m	cs-lin in delta	0.01	-0.20	-0.10	-0.09
123m	cs-SVI	0.06	-0.02	0.01	0.01
123m	cs-lin in strike	0.07	-0.10	-0.01	-0.01
123m	cs-lin in delta	0.00	-0.19	-0.02	-0.07

AL	Diff in Volatility	Max	Min	Median	Mean
1w	cs-SVI	0.02	-0.01	0.00	0.00
1w	cs-lin in strike	0.00	-0.04	-0.03	-0.03
1w	cs-lin in delta	0.22	-0.08	-0.02	0.03
1m	cs-SVI	0.03	-0.01	0.00	0.00
1m	cs-lin in strike	0.00	-0.07	-0.02	-0.03
1m	cs-lin in delta	0.16	-0.09	-0.02	0.00
3m	cs-SVI	0.06	-0.03	0.00	0.01
3m	cs-lin in strike	0.00	-0.10	-0.03	-0.03
3m	cs-lin in delta	0.12	-0.16	-0.04	-0.04
6m	cs-SVI	0.01	-0.02	0.00	0.00
6m	cs-lin in strike	0.03	-0.08	-0.02	-0.02
6m	cs-lin in delta	0.10	-0.19	-0.06	-0.06
9m	cs-SVI	0.06	-0.05	0.01	0.00
9m	cs-lin in strike	0.05	-0.07	-0.02	-0.01
9m	cs-lin in delta	0.05	-0.20	-0.07	-0.08
12m	cs-SVI	0.04	-0.03	0.00	0.00
12m	cs-lin in strike	0.04	-0.07	-0.02	-0.01
12m	cs-lin in delta	0.01	-0.22	-0.08	-0.10
15m	cs-SVI	0.04	-0.03	0.00	0.00
15m	cs-lin in strike	0.04	-0.06	-0.01	-0.01
15m	cs-lin in delta	0.01	-0.21	-0.08	-0.09
21m	cs-SVI	0.05	-0.04	0.00	0.00
21m	cs-lin in strike	0.06	-0.05	-0.01	0.00
21m	cs-lin in delta	0.01	-0.21	-0.08	-0.09
27m	cs-SVI	0.04	-0.04	0.00	0.00
27m	cs-lin in strike	0.06	-0.04	0.00	0.00
27m	cs-lin in delta	0.01	-0.19	-0.08	-0.08
39m	cs-SVI	0.04	-0.03	0.00	0.00
39m	cs-lin in strike	0.07	-0.03	0.00	0.01
39m	cs-lin in delta	0.02	-0.19	-0.08	-0.08
51m	cs-SVI	0.03	-0.03	0.00	0.00
51m	cs-lin in strike	0.06	-0.02	0.00	0.01
51m	cs-lin in delta	0.01	-0.18	-0.07	-0.08
63m	cs-SVI	0.03	-0.03	0.00	-0.01
63m	cs-lin in strike	0.07	-0.02	0.00	0.01
63m	cs-lin in delta	0.01	-0.17	-0.07	-0.07
123m	cs-SVI	0.03	-0.04	0.00	0.00
123m	cs-lin in strike	0.07	-0.01	0.02	0.02
123m	cs-lin in delta	0.00	-0.13	-0.04	-0.04

Zn	Diff in Volatlity	Max	Min	Median	Mean
1w	cs-SVI	0.02	-0.01	0.00	0.00
1w	cs-lin in strike	0.00	-0.09	-0.04	-0.05
1w	cs-lin in delta	0.20	-0.07	0.04	0.05
1m	cs-SVI	0.04	-0.01	0.00	0.01
1m	cs-lin in strike	0.00	-0.12	-0.05	-0.05
1m	cs-lin in delta	0.19	-0.12	0.00	0.00
3m	cs-SVI	0.05	-0.01	0.00	0.01
3m	cs-lin in strike	0.00	-0.14	-0.05	-0.06
3m	cs-lin in delta	0.12	-0.16	-0.05	-0.05
6m	cs-SVI	0.06	-0.02	0.00	0.00
6m	cs-lin in strike	0.00	-0.15	-0.04	-0.06
6m	cs-lin in delta	0.06	-0.20	-0.08	-0.08
9m	cs-SVI	0.03	-0.02	0.00	0.00
9m	cs-lin in strike	0.00	-0.13	-0.03	-0.05
9m	cs-lin in delta	0.02	-0.17	-0.07	-0.08
12m	cs-SVI	0.04	-0.03	0.00	0.00
12m	cs-lin in strike	0.00	-0.14	-0.03	-0.05
12m	cs-lin in delta	0.01	-0.19	-0.09	-0.09
15m	cs-SVI	0.05	-0.01	0.00	0.00
15m	cs-lin in strike	0.00	-0.14	-0.03	-0.05
15m	cs-lin in delta	0.01	-0.20	-0.09	-0.10
21m	cs-SVI	0.04	-0.02	0.00	0.00
21m	cs-lin in strike	0.00	-0.15	-0.02	-0.05
21m	cs-lin in delta	0.01	-0.21	-0.10	-0.10
27m	cs-SVI	0.05	-0.03	0.00	0.00
27m	cs-lin in strike	0.00	-0.08	-0.01	-0.03
27m	cs-lin in delta	0.00	-0.12	-0.06	-0.06
39m	cs-SVI	0.07	-0.03	-0.01	0.00
39m	cs-lin in strike	0.00	-0.09	-0.02	-0.03
39m	cs-lin in delta	0.01	-0.13	-0.06	-0.06
51m	cs-SVI	0.03	-0.01	0.00	0.00
51m	cs-lin in strike	0.00	-0.09	-0.02	-0.03
51m	cs-lin in delta	0.01	-0.14	-0.06	-0.06
63m	cs-SVI	0.03	-0.01	0.00	0.00
63m	cs-lin in strike	0.01	-0.09	-0.02	-0.03
63m	cs-lin in delta	0.00	-0.15	-0.07	-0.07
123m	cs-SVI	0.03	-0.02	0.00	0.00
123m	cs-lin in strike	0.03	-0.11	0.00	-0.02
123m	cs-lin in delta	0.00	-0.18	-0.07	-0.07

Appendix 2B - Percentage Delta Differences

Cu	%diffs in delta	Max	Min	Median	Mean
1w	cs-lin in delta	1.92%	-0.46%	-0.03%	0.09%
1w	cs-lin in strike	0.10%	-0.38%	0.00%	-0.07%
1w	cs-svi	0.24%	-0.05%	0.00%	0.03%
1m	cs-lin in delta	1.40%	-0.41%	-0.02%	0.01%
1m	cs-lin in strike	0.10%	-0.58%	-0.04%	-0.14%
1m	cs-svi	0.13%	-0.03%	0.00%	0.02%
3m	cs-lin in delta	1.21%	-0.89%	-0.04%	-0.17%
3m	cs-lin in strike	0.09%	-0.72%	-0.11%	-0.22%
3m	cs-svi	0.03%	-0.03%	0.00%	0.00%
6m	cs-lin in delta	0.86%	-1.65%	-0.03%	-0.30%
6m	cs-lin in strike	0.09%	-0.86%	-0.04%	-0.25%
6m	cs-svi	0.04%	-0.10%	0.00%	-0.01%
9m	cs-lin in delta	0.58%	-2.19%	-0.01%	-0.33%
9m	cs-lin in strike	0.09%	-0.94%	-0.01%	-0.23%
9m	cs-svi	0.04%	-0.14%	0.00%	-0.01%
12m	cs-lin in delta	0.26%	-1.92%	-0.01%	-0.22%
12m	cs-lin in strike	0.08%	-0.98%	0.00%	-0.19%
12m	cs-svi	0.04%	-0.11%	0.00%	-0.01%
15m	cs-lin in delta	0.25%	-2.63%	0.00%	-0.29%
15m	cs-lin in strike	0.08%	-1.03%	0.00%	-0.18%
15m	cs-svi	0.04%	-0.21%	0.00%	-0.02%
21m	cs-lin in delta	0.19%	-2.75%	0.00%	-0.32%
21m	cs-lin in strike	0.07%	-1.16%	0.00%	-0.22%
21m	cs-svi	0.03%	-0.24%	0.00%	-0.02%
27m	cs-lin in delta	0.16%	-2.78%	-0.02%	-0.40%
27m	cs-lin in strike	0.06%	-1.24%	-0.02%	-0.29%
27m	cs-svi	0.02%	-0.23%	0.00%	-0.02%
39m	cs-lin in delta	0.25%	-1.38%	-0.03%	-0.34%
39m	cs-lin in strike	0.05%	-1.36%	-0.03%	-0.32%
39m	cs-svi	0.01%	-0.06%	-0.01%	-0.01%
51m	cs-lin in delta	0.15%	-1.20%	-0.19%	-0.40%
51m	cs-lin in strike	0.03%	-1.15%	-0.13%	-0.36%
51m	cs-svi	0.03%	-0.05%	0.00%	0.00%
63m	cs-lin in delta	0.08%	-0.92%	-0.20%	-0.33%
63m	cs-lin in strike	0.02%	-0.87%	-0.14%	-0.30%
63m	cs-svi	0.03%	-0.02%	0.00%	0.00%
123m	cs-lin in delta	0.04%	-1.03%	-0.49%	-0.46%
123m	cs-lin in strike	0.11%	-0.75%	-0.01%	-0.21%
123m	cs-svi	0.05%	-0.11%	0.00%	-0.01%

Ni	%diff in delta	Max	Min	Median	Mean
1w	cs-lin in delta	1.32%	-0.20%	0.00%	0.14%
1w	cs-lin in strike	0.05%	-0.21%	-0.01%	-0.04%
1w	cs-svi	0.07%	-0.02%	0.00%	0.00%
1m	cs-lin in delta	1.18%	-0.20%	0.00%	0.09%
1m	cs-lin in strike	0.05%	-0.25%	-0.04%	-0.07%
1m	cs-svi	0.05%	-0.03%	0.00%	0.00%
3m	cs-lin in delta	0.77%	-0.30%	-0.01%	-0.04%
3m	cs-lin in strike	0.05%	-0.27%	-0.08%	-0.09%
3m	cs-svi	0.01%	-0.03%	0.00%	0.00%
6m	cs-lin in delta	0.58%	-0.66%	-0.09%	-0.17%
6m	cs-lin in strike	0.04%	-0.29%	-0.09%	-0.10%
6m	cs-svi	0.01%	-0.05%	0.00%	-0.01%
9m	cs-lin in delta	0.37%	-0.90%	-0.12%	-0.24%
9m	cs-lin in strike	0.03%	-0.31%	-0.07%	-0.09%
9m	cs-svi	0.02%	-0.07%	0.00%	-0.01%
12m	cs-lin in delta	0.16%	-1.13%	-0.17%	-0.30%
12m	cs-lin in strike	0.03%	-0.32%	-0.08%	-0.10%
12m	cs-svi	0.05%	-0.18%	0.00%	-0.02%
15m	cs-lin in delta	0.17%	-1.19%	-0.33%	-0.40%
15m	cs-lin in strike	0.02%	-0.34%	-0.10%	-0.11%
15m	cs-svi	0.02%	-0.09%	-0.01%	-0.02%
21m	cs-lin in delta	0.10%	-1.32%	-0.40%	-0.47%
21m	cs-lin in strike	0.02%	-0.38%	-0.09%	-0.11%
21m	cs-svi	0.02%	-0.12%	-0.01%	-0.03%
27m	cs-lin in delta	0.10%	-1.26%	-0.44%	-0.48%
27m	cs-lin in strike	0.01%	-0.35%	-0.04%	-0.09%
27m	cs-svi	0.02%	-0.11%	-0.01%	-0.03%
39m	cs-lin in delta	0.12%	-0.80%	-0.33%	-0.33%
39m	cs-lin in strike	0.08%	-0.19%	0.00%	-0.02%
39m	cs-svi	0.08%	-0.19%	0.00%	-0.03%
51m	cs-lin in delta	0.08%	-0.77%	-0.35%	-0.34%
51m	cs-lin in strike	0.15%	-0.18%	0.00%	0.00%
51m	cs-svi	0.09%	-0.21%	0.00%	-0.04%
63m	cs-lin in delta	0.06%	-0.82%	-0.43%	-0.39%
63m	cs-lin in strike	0.21%	-0.29%	0.00%	-0.02%
63m	cs-svi	0.19%	-0.24%	0.00%	-0.01%
123m	cs-lin in delta	0.03%	-0.70%	-0.13%	-0.25%
123m	cs-lin in strike	0.66%	-0.36%	0.01%	0.07%
123m	cs-svi	0.21%	-0.09%	0.06%	0.06%

AL	%diff in delta	Max	Min	Median	Mean
1w	cs-lin in delta	1.26%	-0.02%	0.05%	0.28%
1w	cs-lin in strike	0.03%	-0.11%	-0.01%	-0.03%
1w	cs-svi	0.06%	-0.03%	0.00%	0.00%
1m	cs-lin in delta	1.05%	-0.11%	0.04%	0.17%
1m	cs-lin in strike	0.05%	-0.13%	-0.02%	-0.03%
1m	cs-svi	0.06%	-0.04%	0.00%	0.01%
3m	cs-lin in delta	0.95%	-0.28%	0.02%	0.02%
3m	cs-lin in strike	0.06%	-0.16%	0.01%	-0.01%
3m	cs-svi	0.09%	-0.06%	0.00%	0.01%
6m	cs-lin in delta	0.84%	-0.66%	-0.02%	-0.12%
6m	cs-lin in strike	0.21%	-0.13%	0.02%	0.03%
6m	cs-svi	0.00%	-0.12%	-0.01%	-0.03%
9m	cs-lin in delta	0.49%	-0.93%	-0.11%	-0.23%
9m	cs-lin in strike	0.32%	-0.11%	0.02%	0.06%
9m	cs-svi	0.19%	-0.10%	-0.01%	0.02%
12m	cs-lin in delta	0.12%	-1.31%	-0.23%	-0.38%
12m	cs-lin in strike	0.29%	-0.14%	0.02%	0.05%
12m	cs-svi	0.14%	-0.10%	-0.01%	0.01%
15m	cs-lin in delta	0.07%	-1.30%	-0.26%	-0.39%
15m	cs-lin in strike	0.30%	-0.13%	0.02%	0.05%
15m	cs-svi	0.12%	-0.09%	-0.01%	0.00%
21m	cs-lin in delta	0.10%	-1.26%	-0.31%	-0.40%
21m	cs-lin in strike	0.42%	-0.10%	0.02%	0.09%
21m	cs-svi	0.14%	-0.10%	-0.01%	0.00%
27m	cs-lin in delta	0.11%	-1.22%	-0.33%	-0.40%
27m	cs-lin in strike	0.47%	-0.07%	0.01%	0.11%
27m	cs-svi	0.14%	-0.09%	-0.02%	0.00%
39m	cs-lin in delta	0.21%	-1.14%	-0.35%	-0.40%
39m	cs-lin in strike	0.56%	-0.05%	0.02%	0.14%
39m	cs-svi	0.13%	-0.09%	-0.02%	0.00%
51m	cs-lin in delta	0.14%	-1.19%	-0.31%	-0.37%
51m	cs-lin in strike	0.57%	-0.05%	0.01%	0.11%
51m	cs-svi	0.09%	-0.10%	-0.01%	-0.01%
63m	cs-lin in delta	0.09%	-1.13%	-0.32%	-0.36%
63m	cs-lin in strike	0.64%	-0.02%	0.01%	0.12%
63m	cs-svi	0.11%	-0.11%	-0.02%	-0.02%
123m	cs-lin in delta	0.05%	-0.88%	-0.27%	-0.33%
123m	cs-lin in strike	0.99%	0.00%	0.10%	0.25%
123m	cs-svi	0.22%	-0.18%	-0.01%	0.01%

Zn	%diff in delta	Max	Min	Median	Mean
1w	cs-lin in delta	0.56%	-0.09%	0.00%	0.07%
1w	cs-lin in strike	0.02%	-0.10%	-0.01%	-0.02%
1w	cs-svi	0.04%	-0.01%	0.00%	0.00%
1m	cs-lin in delta	0.73%	-0.10%	0.00%	0.07%
1m	cs-lin in strike	0.03%	-0.14%	-0.03%	-0.04%
1m	cs-svi	0.04%	-0.02%	0.00%	0.00%
3m	cs-lin in delta	0.56%	-0.19%	-0.01%	-0.02%
3m	cs-lin in strike	0.03%	-0.17%	-0.04%	-0.06%
3m	cs-svi	0.01%	-0.02%	0.00%	0.00%
6m	cs-lin in delta	0.33%	-0.47%	-0.06%	-0.12%
6m	cs-lin in strike	0.03%	-0.21%	-0.06%	-0.07%
6m	cs-svi	0.01%	-0.03%	0.00%	-0.01%
9m	cs-lin in delta	0.14%	-0.55%	-0.11%	-0.16%
9m	cs-lin in strike	0.02%	-0.21%	-0.07%	-0.07%
9m	cs-svi	0.03%	-0.08%	0.00%	-0.01%
12m	cs-lin in delta	0.04%	-0.76%	-0.19%	-0.24%
12m	cs-lin in strike	0.02%	-0.24%	-0.08%	-0.08%
12m	cs-svi	0.05%	-0.14%	0.00%	-0.02%
15m	cs-lin in delta	0.07%	-0.83%	-0.21%	-0.27%
15m	cs-lin in strike	0.02%	-0.26%	-0.08%	-0.09%
15m	cs-svi	0.01%	-0.05%	0.00%	-0.01%
21m	cs-lin in delta	0.08%	-0.95%	-0.28%	-0.33%
21m	cs-lin in strike	0.02%	-0.30%	-0.08%	-0.10%
21m	cs-svi	0.03%	-0.11%	-0.01%	-0.02%
27m	cs-lin in delta	0.02%	-0.54%	-0.18%	-0.20%
27m	cs-lin in strike	0.01%	-0.18%	-0.04%	-0.06%
27m	cs-svi	0.08%	-0.15%	0.00%	-0.02%
39m	cs-lin in delta	0.08%	-0.61%	-0.22%	-0.23%
39m	cs-lin in strike	0.01%	-0.22%	-0.03%	-0.06%
39m	cs-svi	0.11%	-0.23%	0.00%	-0.04%
51m	cs-lin in delta	0.06%	-0.69%	-0.22%	-0.23%
51m	cs-lin in strike	0.01%	-0.25%	0.00%	-0.07%
51m	cs-svi	0.01%	-0.05%	-0.01%	-0.01%
63m	cs-lin in delta	0.05%	-0.73%	-0.27%	-0.28%
63m	cs-lin in strike	0.05%	-0.28%	0.00%	-0.07%
63m	cs-svi	0.01%	-0.05%	-0.01%	-0.01%
123m	cs-lin in delta	0.04%	-0.91%	-0.48%	-0.44%
123m	cs-lin in strike	0.26%	-0.42%	0.00%	-0.01%
123m	cs-svi	0.04%	-0.09%	-0.01%	-0.01%

Appendix 2C - Percentage Call Premium Differences

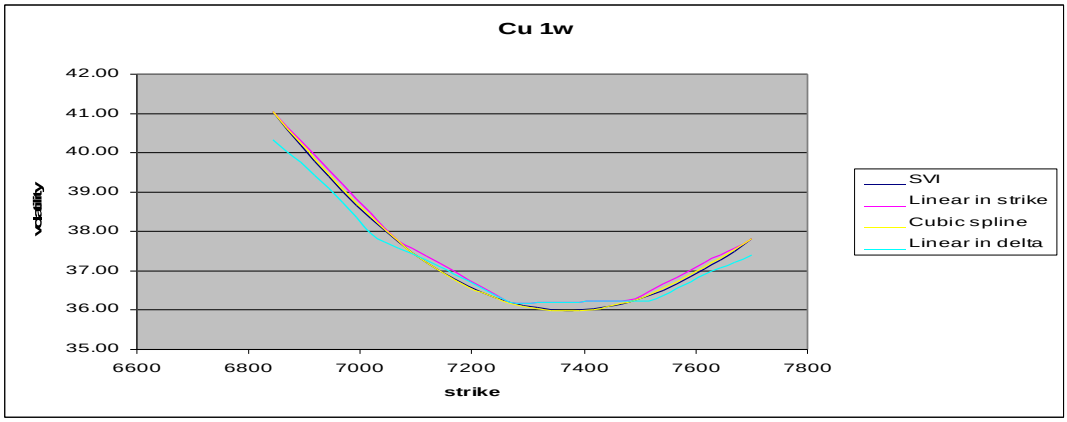
Cu	%diffs in call premiums	Max	Min	Median	Mean
1w	Call prem cs-lin in delta	3.39%	-0.92%	0.22%	0.30%
1w	Call prem cs-lin in strike	0.00%	-0.94%	-0.34%	-0.35%
1w	Call prem cs-svi	0.48%	-0.06%	0.01%	0.07%
1w	Call prem cs-flat	13.39%	-0.80%	1.80%	2.21%
1m	Call prem cs-lin in delta	2.35%	-1.35%	-0.01%	-0.10%
1m	Call prem cs-lin in strike	0.00%	-1.37%	-0.42%	-0.52%
1m	Call prem cs-svi	0.23%	-0.01%	0.01%	0.04%
1m	Call prem cs-flat	19.82%	-0.89%	2.19%	3.24%
3m	Call prem cs-lin in delta	0.64%	-1.82%	-0.46%	-0.66%
3m	Call prem cs-lin in strike	0.00%	-1.81%	-0.48%	-0.67%
3m	Call prem cs-svi	0.06%	-0.03%	0.01%	0.01%
3m	Call prem cs-flat	14.80%	-1.36%	2.15%	2.37%
6m	Call prem cs-lin in delta	1.31%	-2.83%	-0.52%	-0.91%
6m	Call prem cs-lin in strike	0.00%	-2.20%	-0.48%	-0.78%
6m	Call prem cs-svi	0.10%	-0.18%	0.02%	0.01%
6m	Call prem cs-flat	29.09%	-1.80%	1.84%	2.40%
9m	Call prem cs-lin in delta	0.86%	-3.67%	-0.46%	-0.97%
9m	Call prem cs-lin in strike	0.00%	-2.34%	-0.42%	-0.73%
9m	Call prem cs-svi	0.10%	-0.23%	0.02%	0.00%
9m	Call prem cs-flat	31.04%	-1.91%	2.04%	2.23%
12m	Call prem cs-lin in delta	0.38%	-3.36%	-0.38%	-0.80%
12m	Call prem cs-lin in strike	0.00%	-2.44%	-0.34%	-0.66%
12m	Call prem cs-svi	0.11%	-0.19%	0.03%	0.02%
12m	Call prem cs-flat	31.58%	-2.09%	1.58%	1.34%
15m	Call prem cs-lin in delta	0.36%	-4.26%	-0.36%	-0.87%
15m	Call prem cs-lin in strike	0.00%	-2.50%	-0.30%	-0.62%
15m	Call prem cs-svi	0.09%	-0.33%	0.02%	0.00%
15m	Call prem cs-flat	31.28%	-2.11%	1.89%	1.96%
21m	Call prem cs-lin in delta	0.28%	-4.45%	-0.36%	-0.93%
21m	Call prem cs-lin in strike	0.00%	-2.71%	-0.29%	-0.69%
21m	Call prem cs-svi	0.11%	-0.38%	0.04%	0.01%
21m	Call prem cs-flat	32.61%	-2.24%	1.98%	1.76%
27m	Call prem cs-lin in delta	0.23%	-4.47%	-0.35%	-1.06%
27m	Call prem cs-lin in strike	0.00%	-2.81%	-0.28%	-0.81%
27m	Call prem cs-svi	0.12%	-0.36%	0.04%	0.02%
27m	Call prem cs-flat	33.57%	-2.30%	1.99%	1.49%
39m	Call prem cs-lin in delta	0.35%	-2.97%	-0.32%	-0.95%
39m	Call prem cs-lin in strike	0.00%	-2.92%	-0.24%	-0.87%
39m	Call prem cs-svi	0.13%	-0.09%	0.02%	0.03%
39m	Call prem cs-flat	31.30%	-2.63%	0.95%	0.69%
51m	Call prem cs-lin in delta	0.20%	-2.49%	-0.51%	-0.96%
51m	Call prem cs-lin in strike	0.00%	-2.38%	-0.33%	-0.87%
51m	Call prem cs-svi	0.11%	-0.09%	0.02%	0.02%
51m	Call prem cs-flat	28.25%	-1.86%	0.12%	0.34%
63m	Call prem cs-lin in delta	0.11%	-1.85%	-0.52%	-0.76%
63m	Call prem cs-lin in strike	0.00%	-1.75%	-0.42%	-0.70%
63m	Call prem cs-svi	0.09%	-0.05%	0.02%	0.02%
63m	Call prem cs-flat	20.98%	-1.26%	-0.13%	0.12%
123m	Call prem cs-lin in delta	0.05%	-1.62%	-0.90%	-0.82%
123m	Call prem cs-lin in strike	0.15%	-1.41%	-0.09%	-0.43%
123m	Call prem cs-svi	0.08%	-0.20%	0.01%	-0.02%
123m	Call prem cs-flat	17.13%	-0.74%	0.67%	1.84%

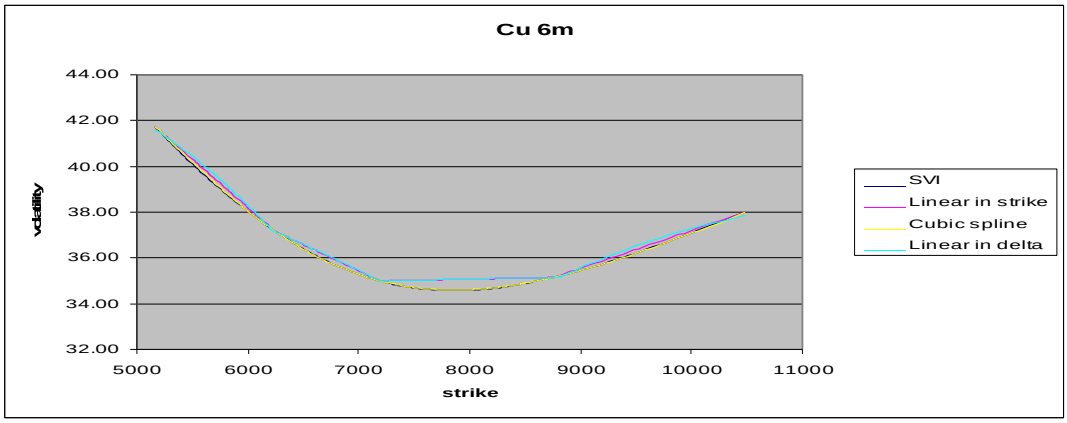
		Max	Min	Median	Mean
Ni	%diffs in call premiums				
1w	Call prem cs-lin in delta	2.36%	-0.40%	0.09%	0.29%
1w	Call prem cs-lin in strike	0.00%	-0.49%	-0.17%	-0.20%
1w	Call prem cs-svi	0.13%	-0.06%	0.01%	0.02%
1w	Call prem cs-flat	12.34%	0.00%	0.81%	2.18%
1m	Call prem cs-lin in delta	1.94%	-0.64%	0.00%	0.09%
1m	Call prem cs-lin in strike	0.00%	-0.67%	-0.23%	-0.26%
1m	Call prem cs-svi	0.09%	-0.02%	0.01%	0.02%
1m	Call prem cs-flat	18.61%	0.00%	0.94%	3.49%
3m	Call prem cs-lin in delta	1.20%	-0.83%	-0.17%	-0.19%
3m	Call prem cs-lin in strike	0.00%	-0.74%	-0.23%	-0.28%
3m	Call prem cs-svi	0.04%	-0.04%	0.00%	0.00%
3m	Call prem cs-flat	22.04%	0.00%	0.88%	4.57%
6m	Call prem cs-lin in delta	0.87%	-1.08%	-0.27%	-0.40%
6m	Call prem cs-lin in strike	0.00%	-0.72%	-0.19%	-0.27%
6m	Call prem cs-svi	0.04%	-0.08%	0.00%	0.00%
6m	Call prem cs-flat	25.39%	-0.01%	0.85%	4.73%
9m	Call prem cs-lin in delta	0.55%	-1.44%	-0.31%	-0.50%
9m	Call prem cs-lin in strike	0.00%	-0.73%	-0.16%	-0.24%
9m	Call prem cs-svi	0.05%	-0.11%	0.00%	-0.01%
9m	Call prem cs-flat	25.98%	-0.01%	0.88%	5.30%
12m	Call prem cs-lin in delta	0.24%	-1.78%	-0.45%	-0.61%
12m	Call prem cs-lin in strike	0.00%	-0.73%	-0.18%	-0.25%
12m	Call prem cs-svi	0.15%	-0.28%	0.01%	-0.02%
12m	Call prem cs-flat	26.47%	-0.01%	0.82%	4.23%
15m	Call prem cs-lin in delta	0.24%	-1.85%	-0.71%	-0.74%
15m	Call prem cs-lin in strike	0.00%	-0.76%	-0.18%	-0.26%
15m	Call prem cs-svi	0.05%	-0.14%	-0.01%	-0.02%
15m	Call prem cs-flat	27.03%	-0.01%	2.73%	6.70%
21m	Call prem cs-lin in delta	0.14%	-2.01%	-0.82%	-0.83%
21m	Call prem cs-lin in strike	0.00%	-0.81%	-0.14%	-0.25%
21m	Call prem cs-svi	0.06%	-0.17%	-0.01%	-0.03%
21m	Call prem cs-flat	27.78%	0.00%	3.73%	7.68%
27m	Call prem cs-lin in delta	0.14%	-1.89%	-0.87%	-0.83%
27m	Call prem cs-lin in strike	0.00%	-0.74%	-0.07%	-0.21%
27m	Call prem cs-svi	0.05%	-0.17%	-0.01%	-0.04%
27m	Call prem cs-flat	26.83%	-0.01%	3.96%	7.35%
39m	Call prem cs-lin in delta	0.16%	-1.17%	-0.62%	-0.56%
39m	Call prem cs-lin in strike	0.12%	-0.39%	-0.02%	-0.07%
39m	Call prem cs-svi	0.17%	-0.27%	-0.01%	-0.03%
39m	Call prem cs-flat	21.69%	-0.05%	3.98%	6.14%
51m	Call prem cs-lin in delta	0.10%	-1.10%	-0.66%	-0.57%
51m	Call prem cs-lin in strike	0.22%	-0.35%	-0.02%	-0.03%
51m	Call prem cs-svi	0.20%	-0.29%	-0.02%	-0.04%
51m	Call prem cs-flat	20.90%	-0.05%	3.93%	5.69%
63m	Call prem cs-lin in delta	0.08%	-1.17%	-0.76%	-0.65%
63m	Call prem cs-lin in strike	0.30%	-0.54%	-0.03%	-0.07%
63m	Call prem cs-svi	0.39%	-0.34%	0.00%	0.01%
63m	Call prem cs-flat	20.61%	0.00%	3.35%	5.02%
123m	Call prem cs-lin in delta	0.03%	-1.15%	-0.17%	-0.42%
123m	Call prem cs-lin in strike	0.89%	-0.59%	-0.04%	0.06%
123m	Call prem cs-svi	0.29%	-0.14%	0.11%	0.09%
123m	Call prem cs-flat	13.94%	-0.01%	2.75%	3.54%

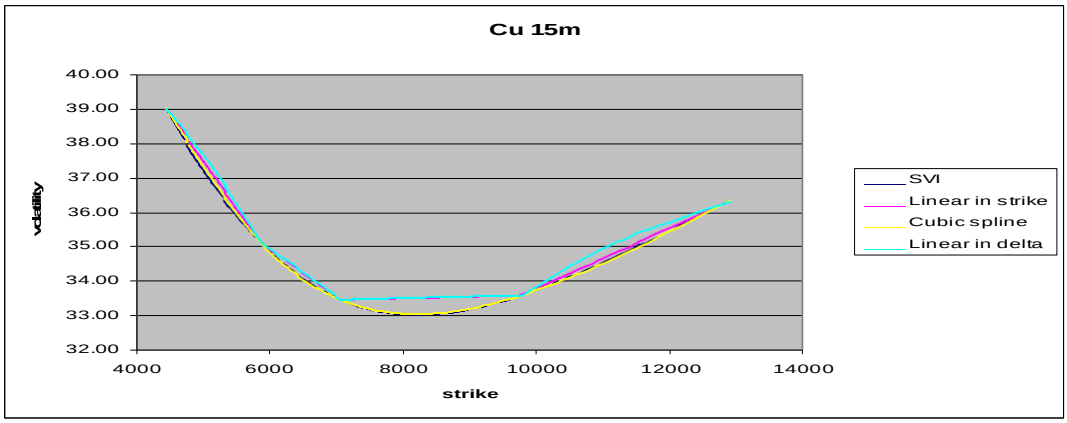
AL	%diffs in call premiums	Max	Min	Median	Mean
1w	Call prem cs-lin in delta	2.43%	-0.19%	-0.02%	0.49%
1w	Call prem cs-lin in strike	0.00%	-0.30%	-0.17%	-0.15%
1w	Call prem cs-svi	0.11%	-0.05%	0.00%	0.01%
1w	Call prem cs-flat	19.30%	-0.85%	0.31%	4.09%
1m	Call prem cs-lin in delta	1.83%	-0.42%	-0.04%	0.22%
1m	Call prem cs-lin in strike	0.00%	-0.44%	-0.16%	-0.17%
1m	Call prem cs-svi	0.11%	-0.05%	0.02%	0.02%
1m	Call prem cs-flat	30.89%	-1.04%	0.83%	6.49%
3m	Call prem cs-lin in delta	1.54%	-0.93%	-0.17%	-0.13%
3m	Call prem cs-lin in strike	0.04%	-0.51%	-0.05%	-0.13%
3m	Call prem cs-svi	0.20%	-0.15%	0.02%	0.02%
3m	Call prem cs-flat	48.36%	-1.46%	3.66%	11.85%
6m	Call prem cs-lin in delta	1.32%	-1.16%	-0.24%	-0.36%
6m	Call prem cs-lin in strike	0.37%	-0.42%	-0.05%	-0.03%
6m	Call prem cs-svi	0.02%	-0.20%	-0.02%	-0.05%
6m	Call prem cs-flat	54.72%	-1.68%	6.51%	14.81%
9m	Call prem cs-lin in delta	0.76%	-1.59%	-0.35%	-0.56%
9m	Call prem cs-lin in strike	0.54%	-0.36%	-0.04%	0.03%
9m	Call prem cs-svi	0.36%	-0.25%	0.02%	0.03%
9m	Call prem cs-flat	55.78%	-1.68%	7.98%	15.82%
12m	Call prem cs-lin in delta	0.18%	-2.20%	-0.68%	-0.81%
12m	Call prem cs-lin in strike	0.49%	-0.41%	-0.04%	0.01%
12m	Call prem cs-svi	0.26%	-0.16%	0.00%	0.01%
12m	Call prem cs-flat	54.79%	-1.40%	8.13%	15.71%
15m	Call prem cs-lin in delta	0.11%	-2.15%	-0.69%	-0.80%
15m	Call prem cs-lin in strike	0.50%	-0.38%	-0.04%	0.02%
15m	Call prem cs-svi	0.23%	-0.16%	0.00%	0.00%
15m	Call prem cs-flat	53.32%	-1.32%	8.37%	15.49%
21m	Call prem cs-lin in delta	0.15%	-2.05%	-0.74%	-0.80%
21m	Call prem cs-lin in strike	0.68%	-0.28%	-0.03%	0.10%
21m	Call prem cs-svi	0.26%	-0.20%	0.00%	0.00%
21m	Call prem cs-flat	53.67%	-1.34%	10.02%	16.43%
27m	Call prem cs-lin in delta	0.16%	-1.95%	-0.77%	-0.79%
27m	Call prem cs-lin in strike	0.76%	-0.20%	-0.01%	0.15%
27m	Call prem cs-svi	0.25%	-0.22%	0.00%	-0.01%
27m	Call prem cs-flat	52.49%	-1.30%	10.80%	16.55%
39m	Call prem cs-lin in delta	0.30%	-1.79%	-0.78%	-0.76%
39m	Call prem cs-lin in strike	0.88%	-0.16%	0.00%	0.20%
39m	Call prem cs-svi	0.22%	-0.22%	-0.01%	-0.02%
39m	Call prem cs-flat	52.00%	-1.17%	12.29%	17.24%
51m	Call prem cs-lin in delta	0.19%	-1.83%	-0.70%	-0.72%
51m	Call prem cs-lin in strike	0.89%	-0.13%	-0.01%	0.15%
51m	Call prem cs-svi	0.15%	-0.20%	-0.02%	-0.03%
51m	Call prem cs-flat	49.93%	-1.00%	8.40%	13.50%
63m	Call prem cs-lin in delta	0.13%	-1.72%	-0.70%	-0.71%
63m	Call prem cs-lin in strike	0.98%	-0.07%	0.00%	0.17%
63m	Call prem cs-svi	0.18%	-0.26%	-0.05%	-0.05%
63m	Call prem cs-flat	49.32%	-1.01%	6.40%	12.44%
123m	Call prem cs-lin in delta	0.08%	-1.35%	-0.54%	-0.58%
123m	Call prem cs-lin in strike	1.34%	-0.03%	0.21%	0.39%
123m	Call prem cs-svi	0.34%	-0.40%	0.02%	0.00%
123m	Call prem cs-flat	49.08%	-1.09%	8.68%	15.02%

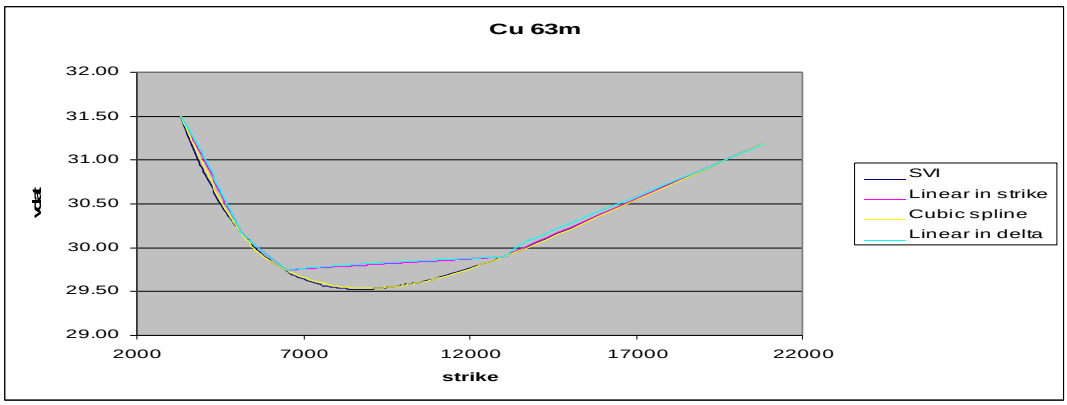
Zn	%diffs in call premiums	Max	Min	Median	Mean
1w	Call prem cs-lin in delta	1.05%	-0.20%	0.02%	0.16%
1w	Call prem cs-lin in strike	0.00%	-0.25%	-0.10%	-0.11%
1w	Call prem cs-svi	0.07%	-0.02%	0.00%	0.01%
1w	Call prem cs-flat	6.26%	-0.01%	0.34%	1.22%
1m	Call prem cs-lin in delta	1.20%	-0.33%	0.00%	0.09%
1m	Call prem cs-lin in strike	0.00%	-0.36%	-0.12%	-0.14%
1m	Call prem cs-svi	0.07%	-0.01%	0.01%	0.01%
1m	Call prem cs-flat	12.08%	-0.02%	0.43%	2.33%
3m	Call prem cs-lin in delta	0.87%	-0.52%	-0.10%	-0.10%
3m	Call prem cs-lin in strike	0.00%	-0.45%	-0.14%	-0.17%
3m	Call prem cs-svi	0.03%	-0.02%	0.01%	0.01%
3m	Call prem cs-flat	16.50%	-0.02%	0.59%	3.48%
6m	Call prem cs-lin in delta	0.49%	-0.77%	-0.19%	-0.28%
6m	Call prem cs-lin in strike	0.00%	-0.53%	-0.14%	-0.19%
6m	Call prem cs-svi	0.03%	-0.04%	0.00%	0.00%
6m	Call prem cs-flat	19.28%	-0.02%	1.15%	4.40%
9m	Call prem cs-lin in delta	0.21%	-0.88%	-0.29%	-0.34%
9m	Call prem cs-lin in strike	0.00%	-0.50%	-0.14%	-0.18%
9m	Call prem cs-svi	0.09%	-0.13%	0.00%	-0.01%
9m	Call prem cs-flat	15.91%	0.00%	0.97%	3.65%
12m	Call prem cs-lin in delta	0.05%	-1.20%	-0.45%	-0.46%
12m	Call prem cs-lin in strike	0.00%	-0.55%	-0.15%	-0.19%
12m	Call prem cs-svi	0.15%	-0.21%	-0.01%	-0.02%
12m	Call prem cs-flat	17.54%	0.00%	1.29%	4.18%
15m	Call prem cs-lin in delta	0.10%	-1.30%	-0.49%	-0.51%
15m	Call prem cs-lin in strike	0.00%	-0.60%	-0.16%	-0.21%
15m	Call prem cs-svi	0.04%	-0.07%	0.00%	-0.01%
15m	Call prem cs-flat	19.02%	0.00%	1.60%	4.67%
21m	Call prem cs-lin in delta	0.12%	-1.46%	-0.58%	-0.60%
21m	Call prem cs-lin in strike	0.00%	-0.65%	-0.14%	-0.21%
21m	Call prem cs-svi	0.07%	-0.16%	-0.01%	-0.03%
21m	Call prem cs-flat	20.54%	0.00%	2.21%	5.34%
27m	Call prem cs-lin in delta	0.03%	-0.82%	-0.36%	-0.35%
27m	Call prem cs-lin in strike	0.00%	-0.38%	-0.07%	-0.12%
27m	Call prem cs-svi	0.19%	-0.22%	-0.01%	-0.01%
27m	Call prem cs-flat	12.06%	0.00%	1.34%	3.13%
39m	Call prem cs-lin in delta	0.12%	-0.91%	-0.44%	-0.41%
39m	Call prem cs-lin in strike	0.00%	-0.45%	-0.04%	-0.14%
39m	Call prem cs-svi	0.27%	-0.35%	-0.02%	-0.04%
39m	Call prem cs-flat	14.29%	0.00%	1.51%	3.34%
51m	Call prem cs-lin in delta	0.08%	-1.02%	-0.43%	-0.43%
51m	Call prem cs-lin in strike	0.00%	-0.50%	-0.04%	-0.16%
51m	Call prem cs-svi	0.03%	-0.07%	-0.01%	-0.01%
51m	Call prem cs-flat	15.50%	0.00%	1.14%	3.20%
63m	Call prem cs-lin in delta	0.07%	-1.07%	-0.54%	-0.49%
63m	Call prem cs-lin in strike	0.07%	-0.55%	-0.04%	-0.16%
63m	Call prem cs-svi	0.03%	-0.08%	-0.01%	-0.01%
63m	Call prem cs-flat	16.51%	0.00%	0.86%	3.47%
123m	Call prem cs-lin in delta	0.05%	-1.32%	-0.81%	-0.72%
123m	Call prem cs-lin in strike	0.38%	-0.78%	0.00%	-0.07%
123m	Call prem cs-svi	0.06%	-0.16%	-0.01%	-0.02%
123m	Call prem cs-flat	21.55%	-0.01%	4.90%	7.07%

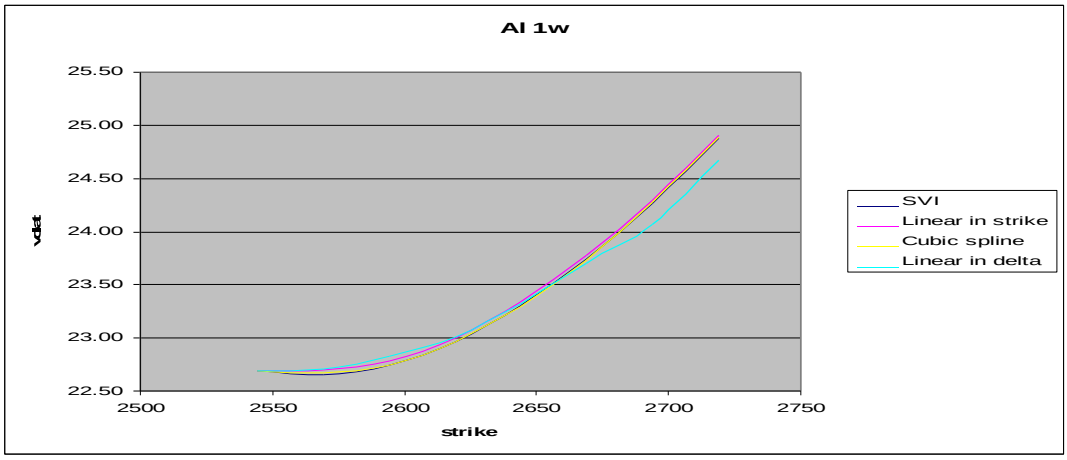
Appendix 3 - Volatility Graphs

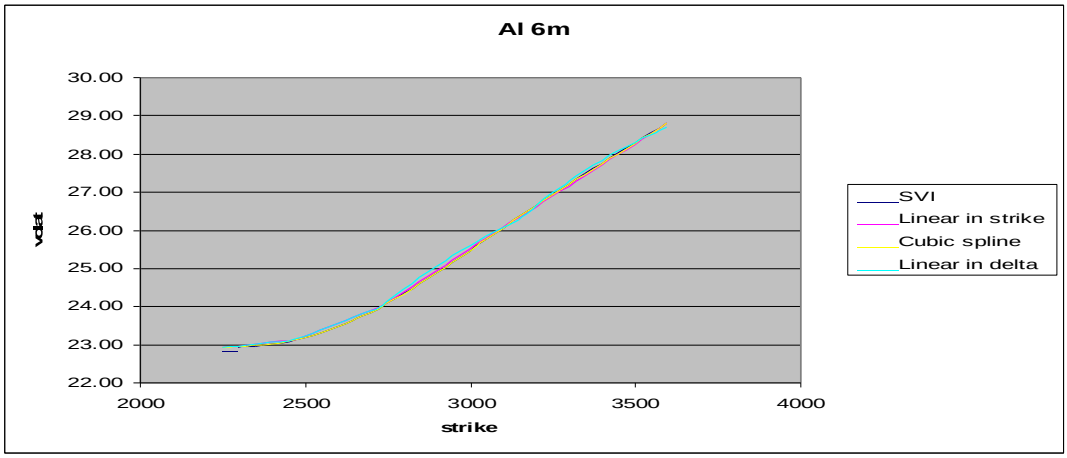


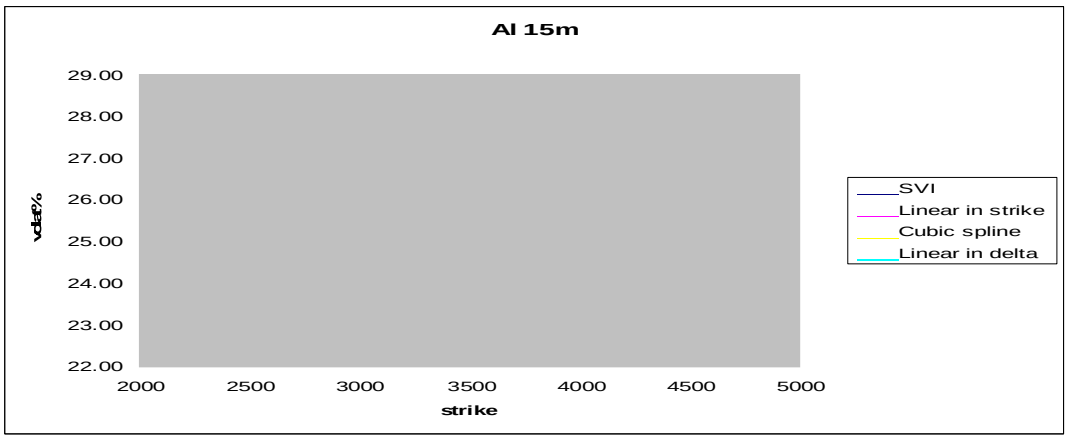


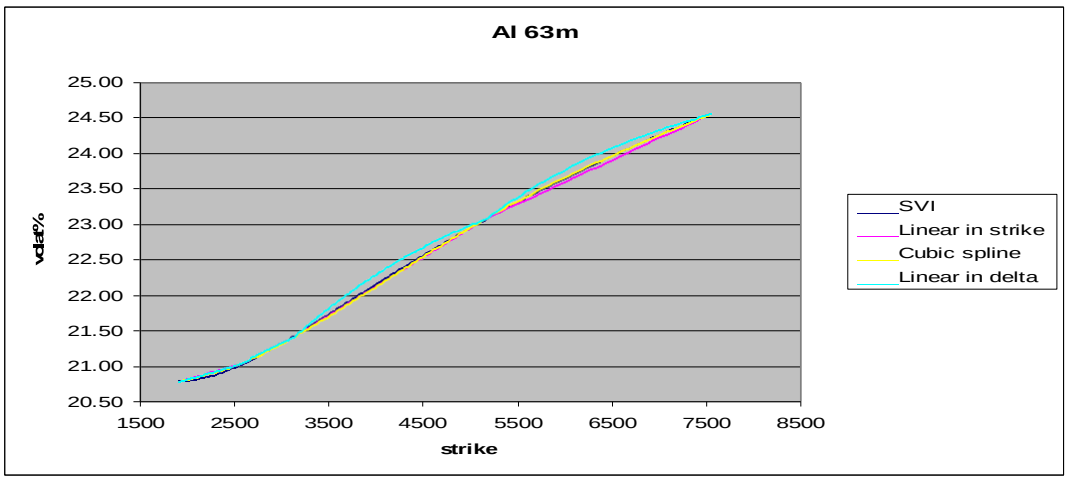


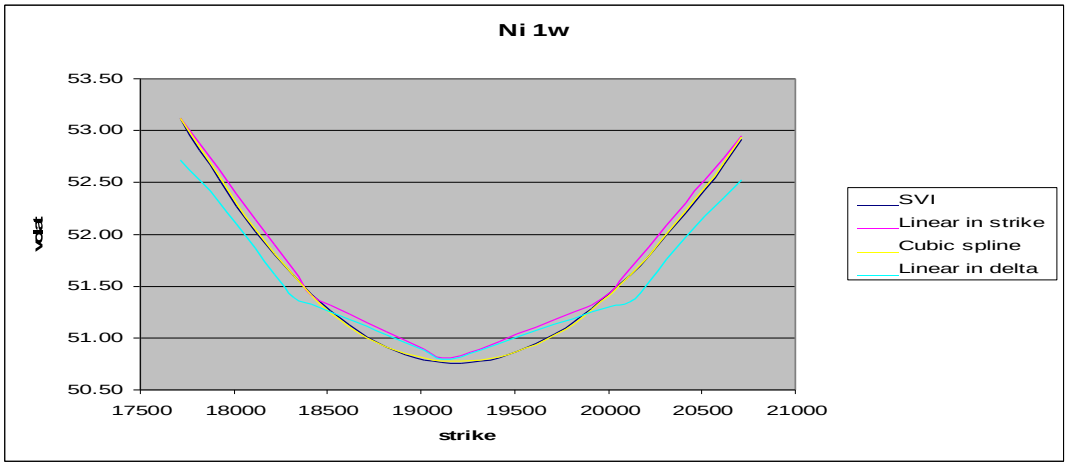


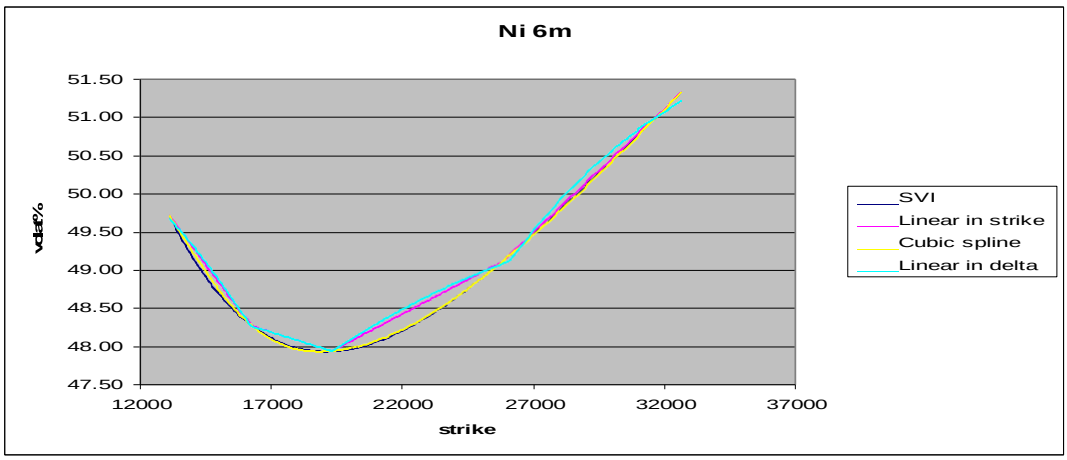

Al 1w
25.50
25.00
24.50
24.00
23.50
23.00
22.50
vdat
2500
2550
2600
2650
2700
2750
strike
SVI
Linear in strike
Cubic spline
Linear in delta

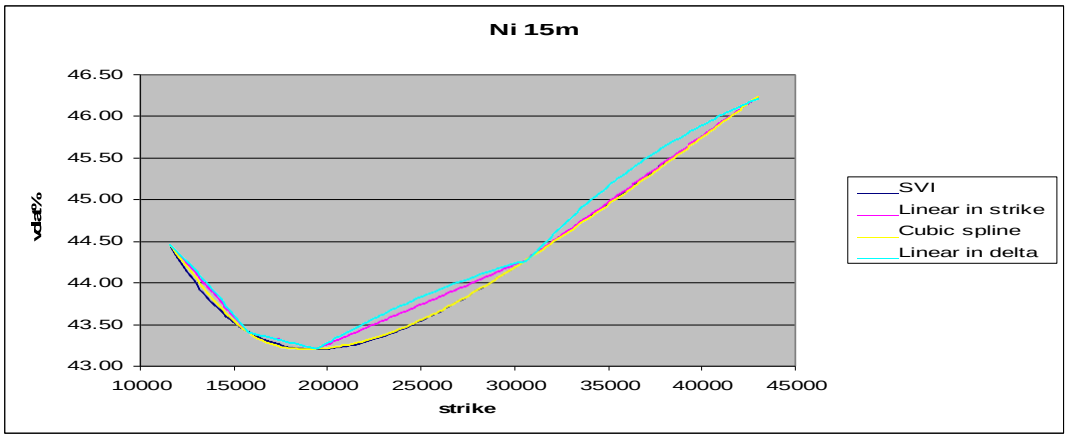

Al 6m
30.00
29.00
28.00
27.00
26.00
25.00
24.00
23.00
22.00
vdat
2000
2500
3000
3500
4000
strike
SVI
Linear in strike
Cubic spline
Linear in delta

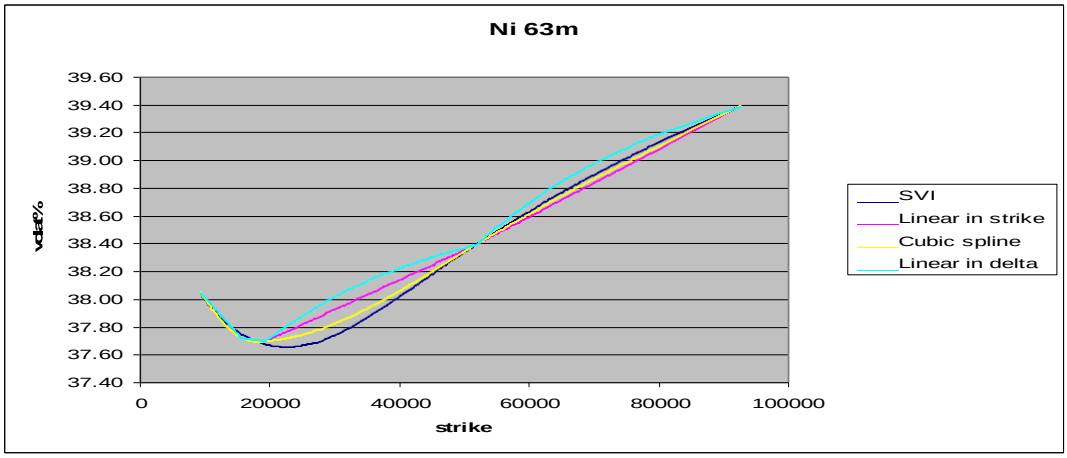

Al 15m
29.00
28.00
27.00
26.00
25.00
24.00
23.00
22.00
vdat%
2000
2500
3000
3500
4000
4500
5000
strike
SVI
Linear in strike
Cubic spline
Linear in delta

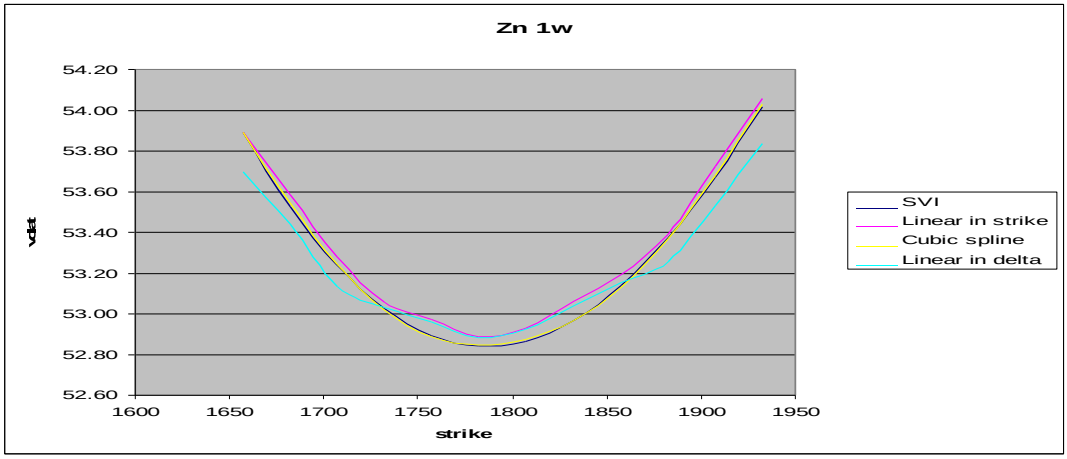

Al 63m
25.00
24.50
24.00
23.50
23.00
22.50
22.00
21.50
21.00
20.50
vdat%
1500
2500
3500
4500
5500
6500
7500
8500
strike
SVI
Linear in strike
Cubic spline
Linear in delta

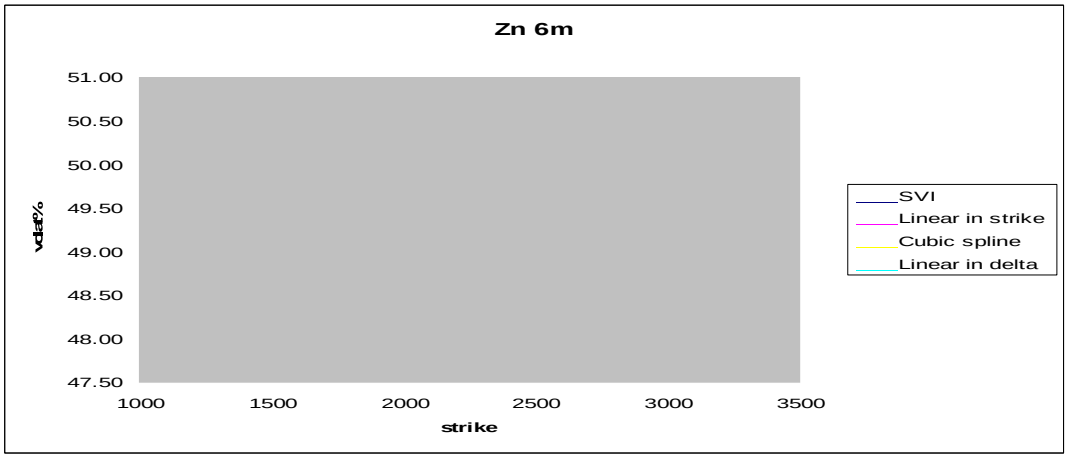

Ni 1w
vdat
53.50
53.00
52.50
52.00
51.50
51.00
50.50
17500
18000
18500
19000
19500
20000
20500
21000
strike
SVI
Linear in strike
Cubic spline
Linear in delta

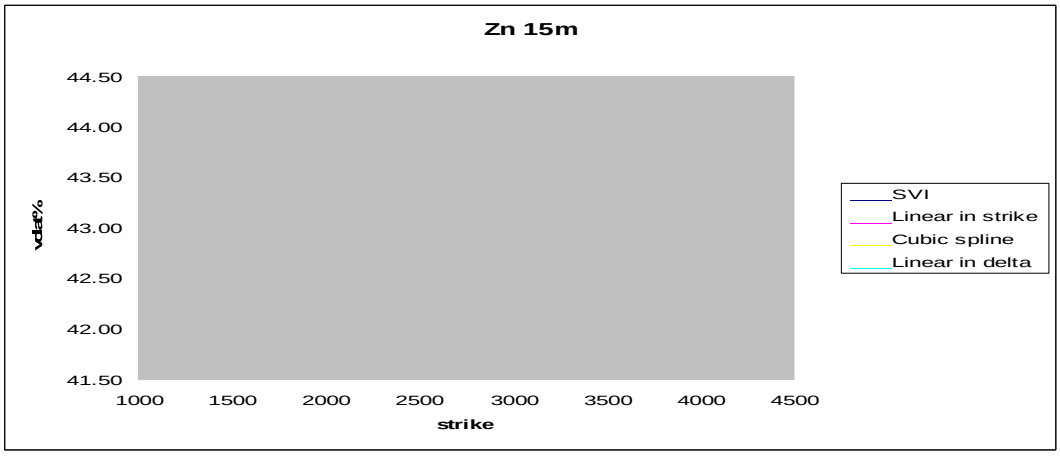

Ni 6m
vdat%
51.50
51.00
50.50
50.00
49.50
49.00
48.50
48.00
47.50
12000
17000
22000
27000
32000
37000
strike
SVI
Linear in strike
Cubic spline
Linear in delta

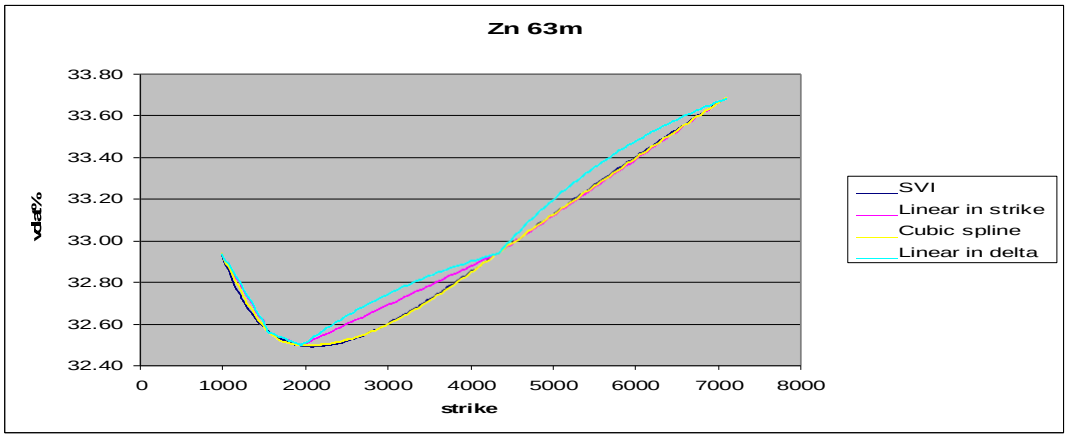

Ni 15m
vdat%
46.50
46.00
45.50
45.00
44.50
44.00
43.50
43.00
10000
15000
20000
25000
30000
35000
40000
45000
strike
SVI
Linear in strike
Cubic spline
Linear in delta


Ni 63m
vdat%
39.60
39.40
39.20
39.00
38.80
38.60
38.40
38.20
38.00
37.80
37.60
37.40
0
20000
40000
60000
80000
100000
strike
SVI
Linear in strike
Cubic spline
Linear in delta

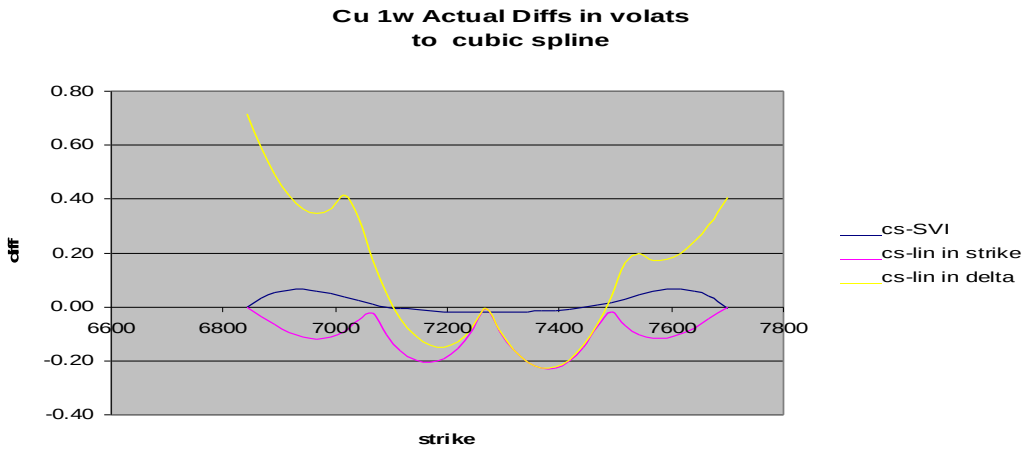

Zn 1w
54.20
54.00
53.80
53.60
53.40
53.20
53.00
52.80
52.60
1600
1650
1700
1750
1800
1850
1900
1950
strike
SVI
Linear in strike
Cubic spline
Linear in delta

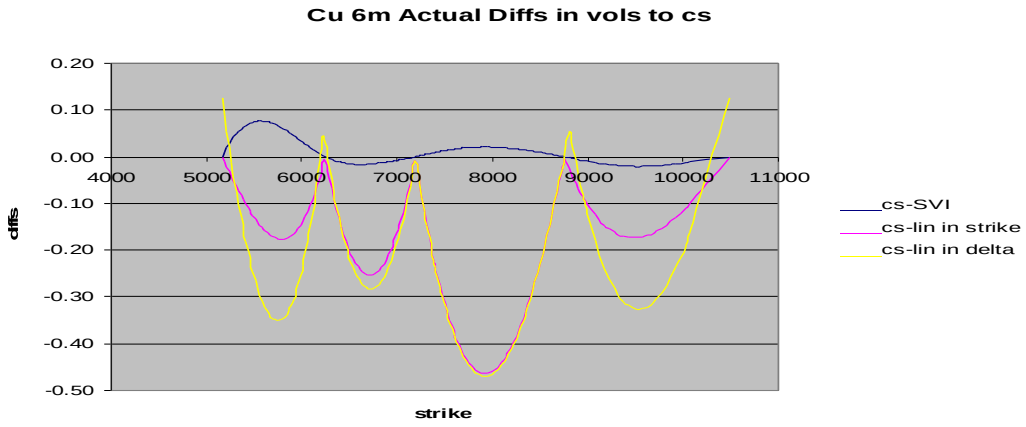

Zn 6m
51.00
50.50
50.00
49.50
49.00
48.50
48.00
47.50
1000
1500
2000
2500
3000
3500
strike
SVI
Linear in strike
Cubic spline
Linear in delta

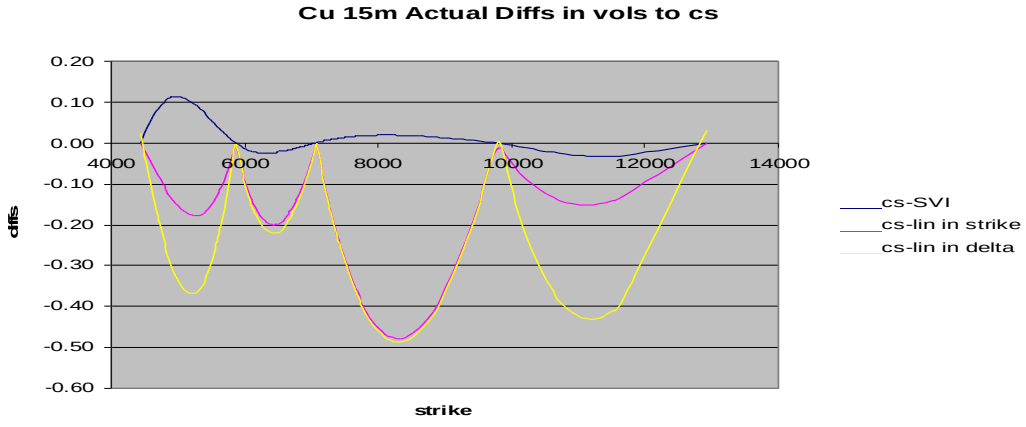

Zn 15m
44.50
44.00
43.50
43.00
42.50
42.00
41.50
1000
1500
2000
2500
3000
3500
4000
4500
strike
SVI
Linear in strike
Cubic spline
Linear in delta

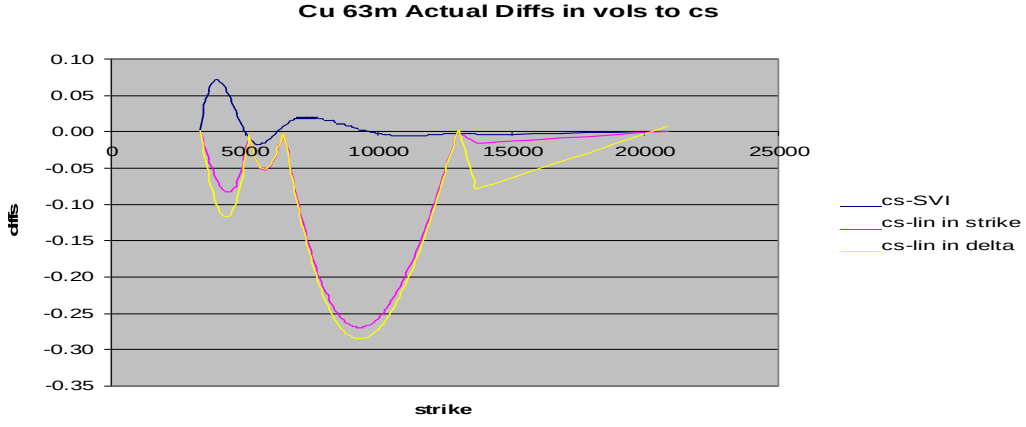

Zn 63m
33.80
33.60
33.40
33.20
33.00
32.80
32.60
32.40
0
1000
2000
3000
4000
5000
6000
7000
8000
strike
SVI
Linear in strike
Cubic spline
Linear in delta

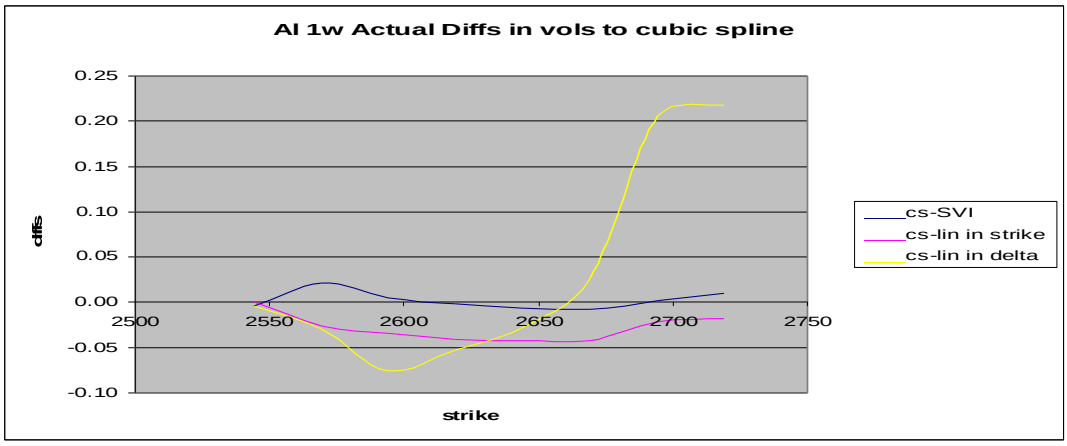
Appendix 3A - Actual Differences in Volatilities to Cubic Spline

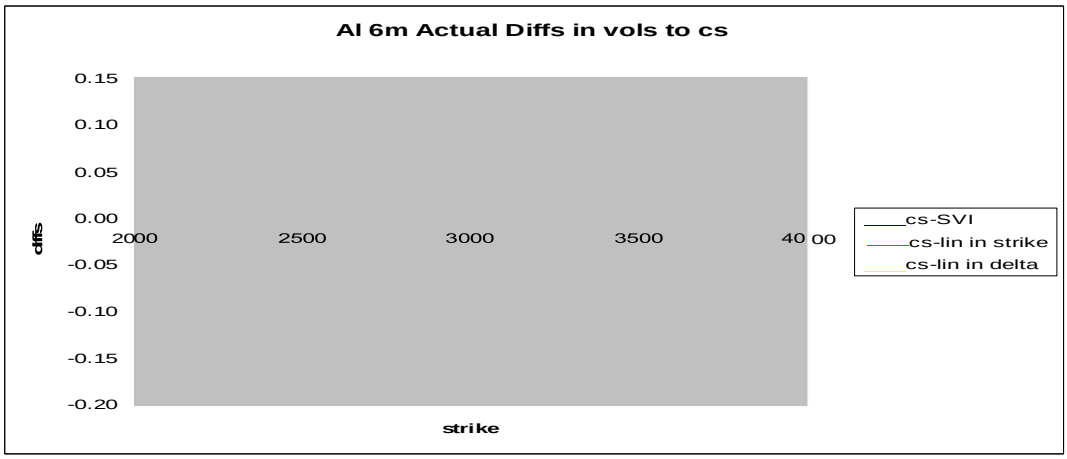


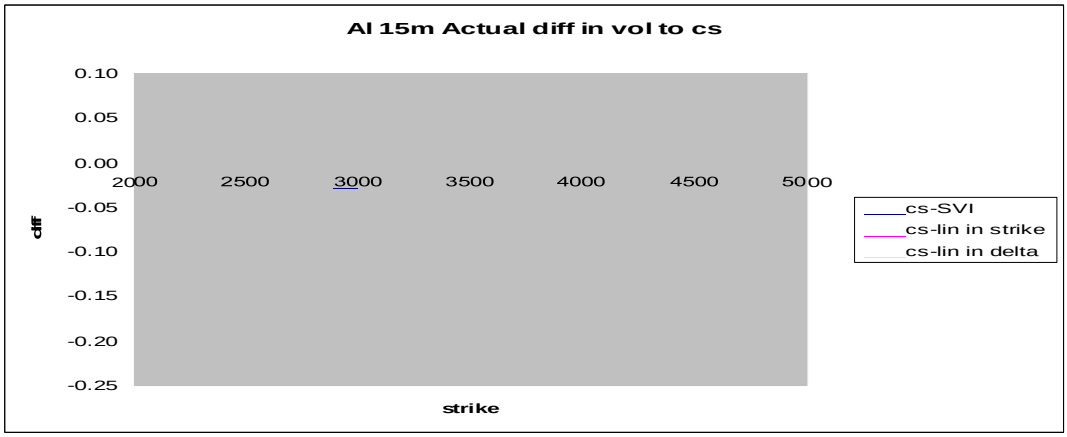


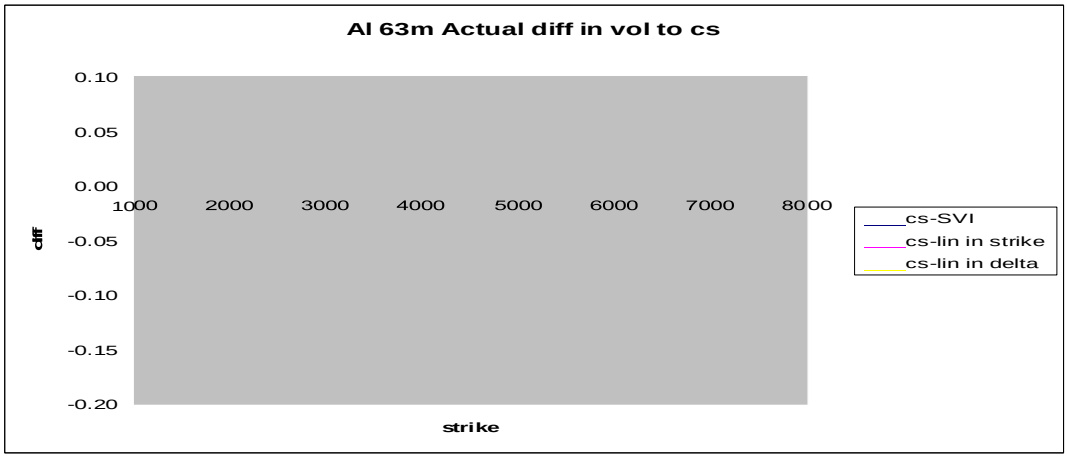


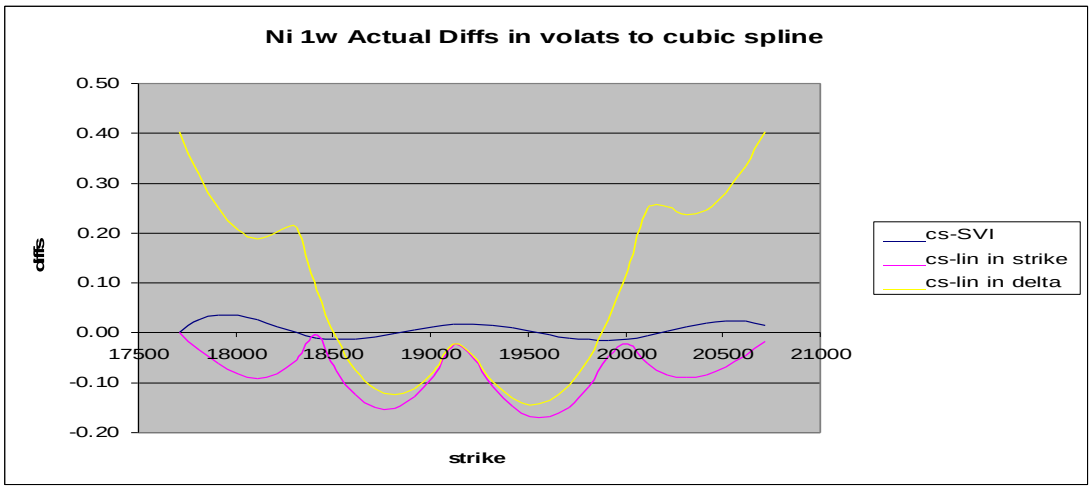


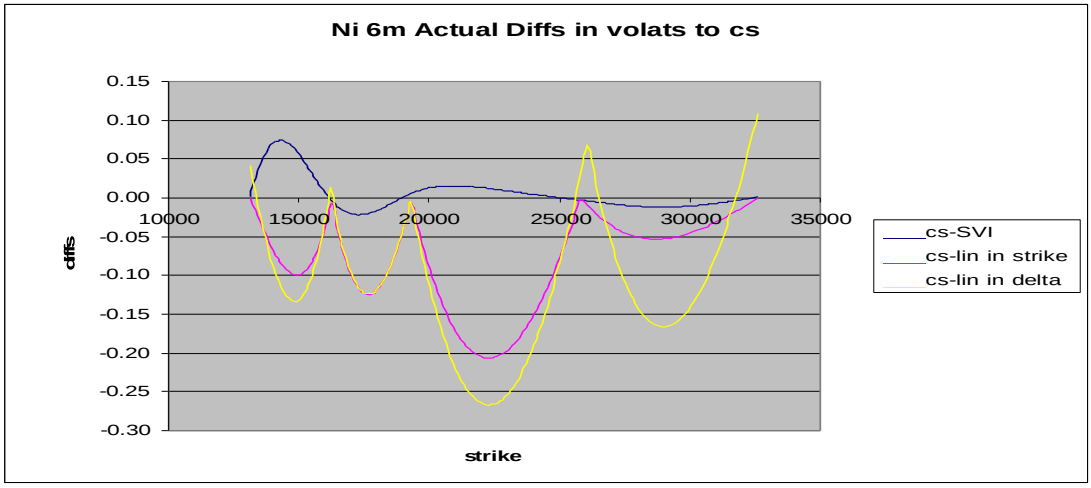

Al 1w Actual Diffs in vols to cubic spline
0.25
0.20
0.15
0.10
0.05
0.00
-0.05
-0.10
diffs
2500
2550
2600
2650
2700
2750
strike
cs-SVI
cs-lin in strike
cs-lin in delta

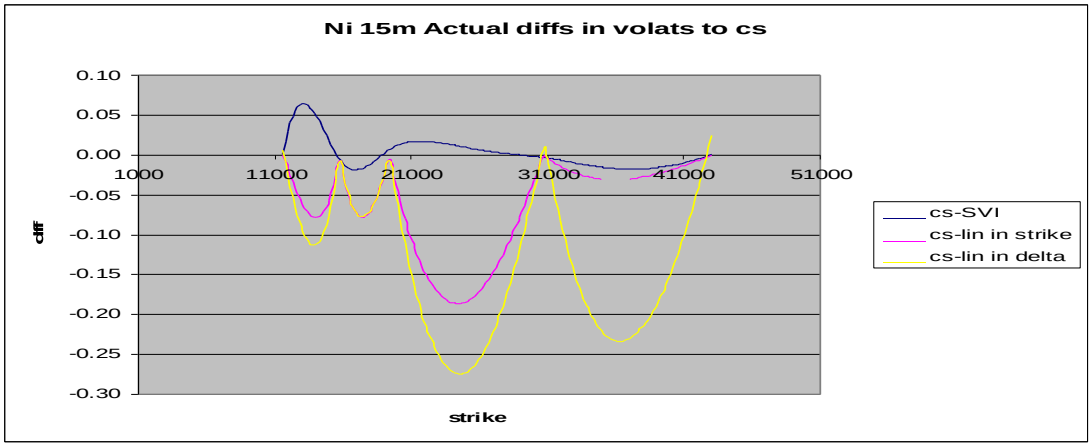

Al 6m Actual Diffs in vols to cs
0.15
0.10
0.05
0.00
-0.05
-0.10
-0.15
-0.20
diffs
2000
2500
3000
3500
40 00
strike
cs-SVI
cs-lin in strike
cs-lin in delta

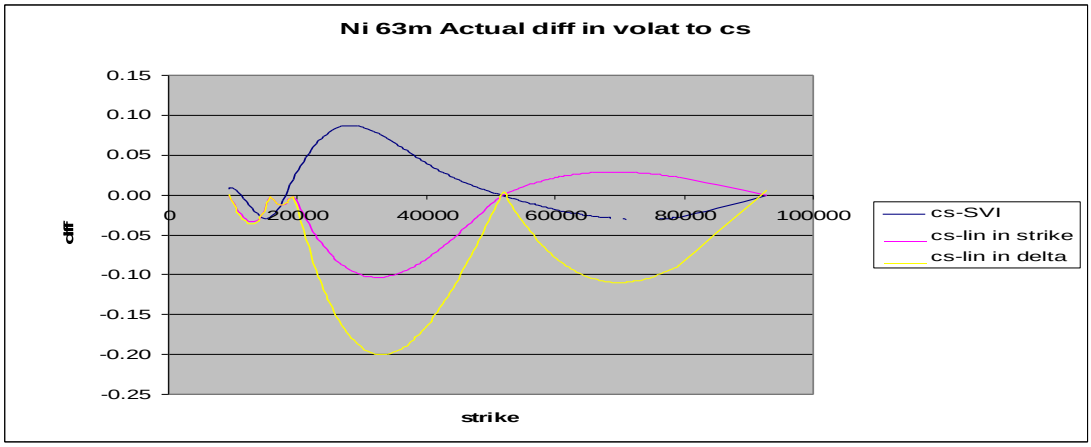

Al 15m Actual diff in vol to cs
0.10
0.05
0.00
-0.05
-0.10
-0.15
-0.20
-0.25
diff
2000
2500
3000
3500
4000
4500
5000
strike
cs-SVI
cs-lin in strike
cs-lin in delta

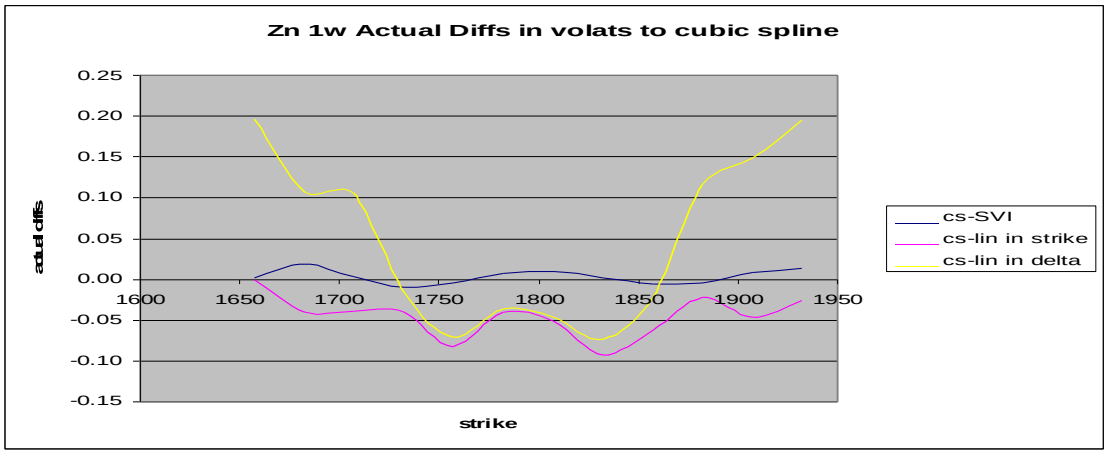

Al 63m Actual diff in vol to cs
0.10
0.05
0.00
-0.05
-0.10
-0.15
-0.20
diff
1000
2000
3000
4000
5000
6000
7000
8000
strike
cs-SVI
cs-lin in strike
cs-lin in delta

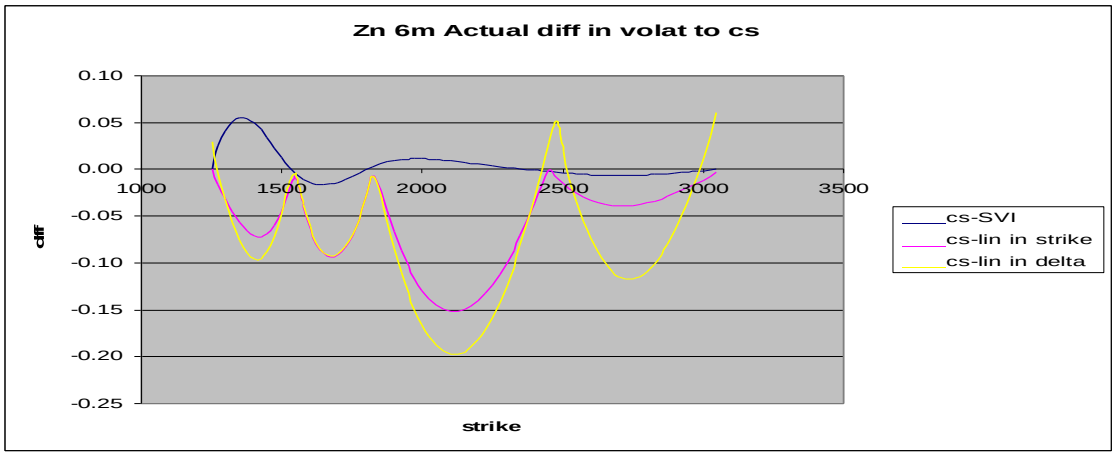

Ni 1w Actual Diffs in volats to cubic spline
0.50
0.40
0.30
0.20
0.10
0.00
-0.10
-0.20
diffs
17500
18000
18500
19000
19500
20000
20500
21000
strike
cs-SVI
cs-lin in strike
cs-lin in delta

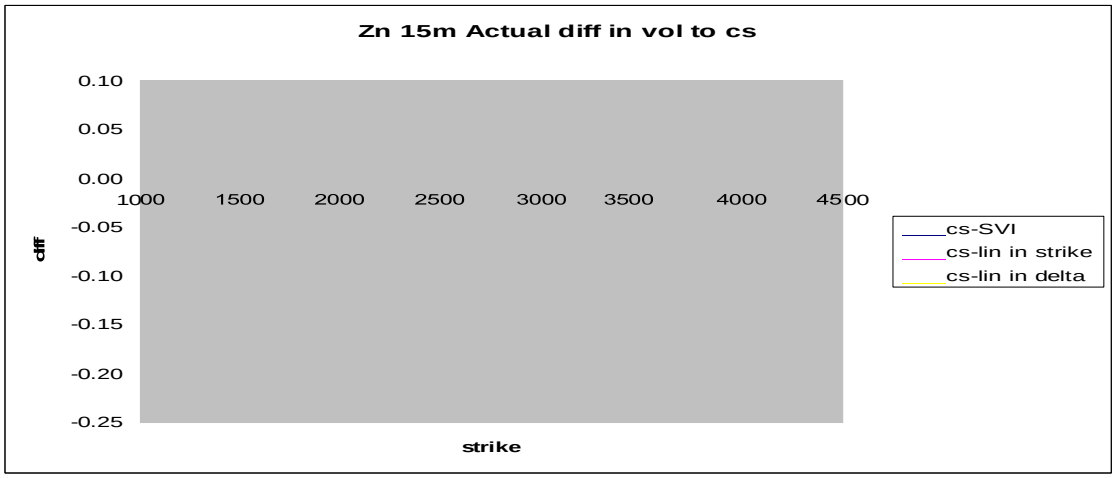

Ni 6m Actual Diffs in volats to cs
0.15
0.10
0.05
0.00
-0.05
-0.10
-0.15
-0.20
-0.25
-0.30
diffs
10000
15000
20000
25000
30000
35000
strike
cs-SVI
cs-lin in strike
cs-lin in delta

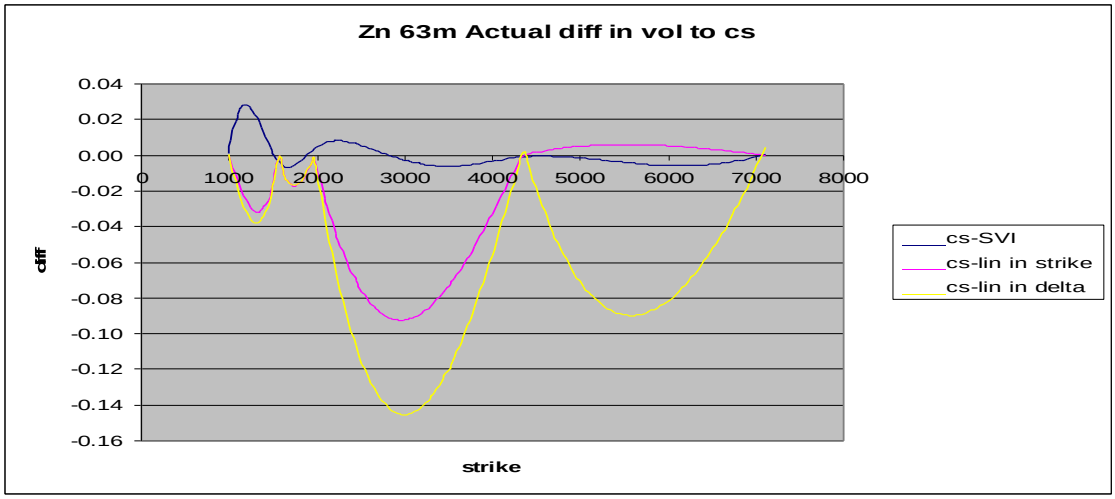

Ni 15m Actual diffs in volats to cs
0.10
0.05
0.00
-0.05
-0.10
-0.15
-0.20
-0.25
-0.30
diff
1000
11000
21000
31000
41000
51000
strike
cs-SVI
cs-lin in strike
cs-lin in delta


Ni 63m Actual diff in volat to cs
0.15
0.10
0.05
0.00
-0.05
-0.10
-0.15
-0.20
-0.25
diff
0
20000
40000
60000
80000
100000
strike
cs-SVI
cs-lin in strike
cs-lin in delta

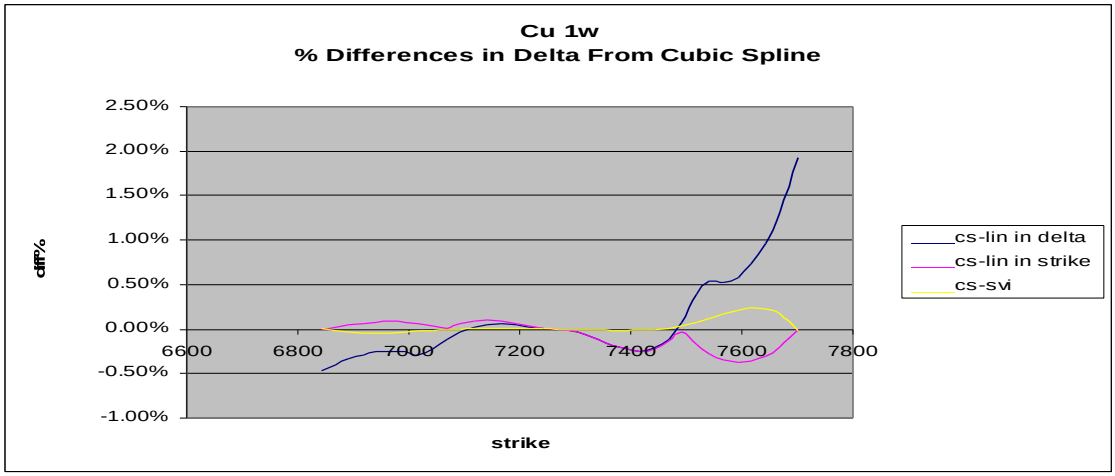

Zn 1w Actual Diffs in volats to cubic spline
0.25
0.20
0.15
0.10
0.05
0.00
-0.05
-0.10
-0.15
1600
1650
1700
1750
1800
1850
1900
1950
actual diffs
strike
cs-SVI
cs-lin in strike
cs-lin in delta

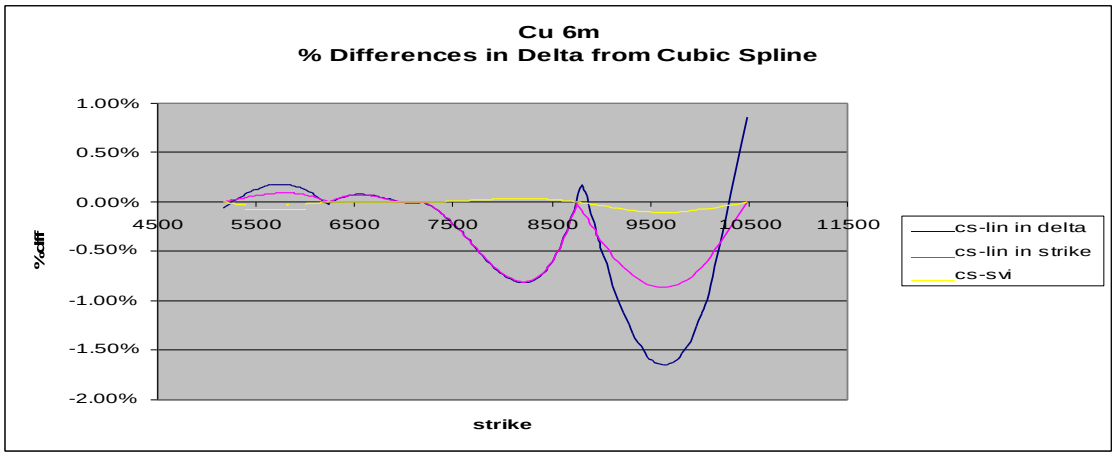

Zn 6m Actual diff in volat to cs
0.10
0.05
0.00
-0.05
-0.10
-0.15
-0.20
-0.25
1000
1500
2000
2500
3000
3500
diff
strike
cs-SVI
cs-lin in strike
cs-lin in delta

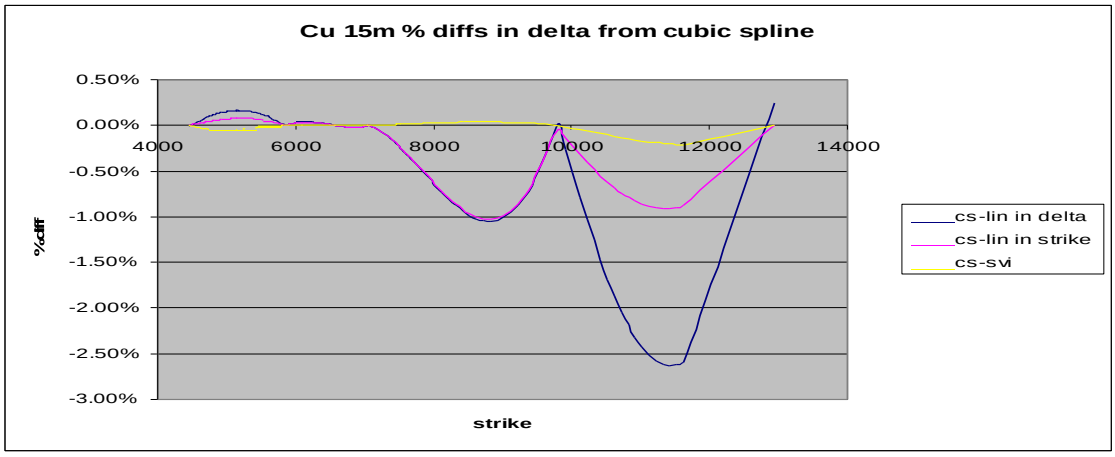

Zn 15m Actual diff in vol to cs
0.10
0.05
0.00
-0.05
-0.10
-0.15
-0.20
-0.25
1000
1500
2000
2500
3000
3500
4000
4500
diff
strike
cs-SVI
cs-lin in strike
cs-lin in delta

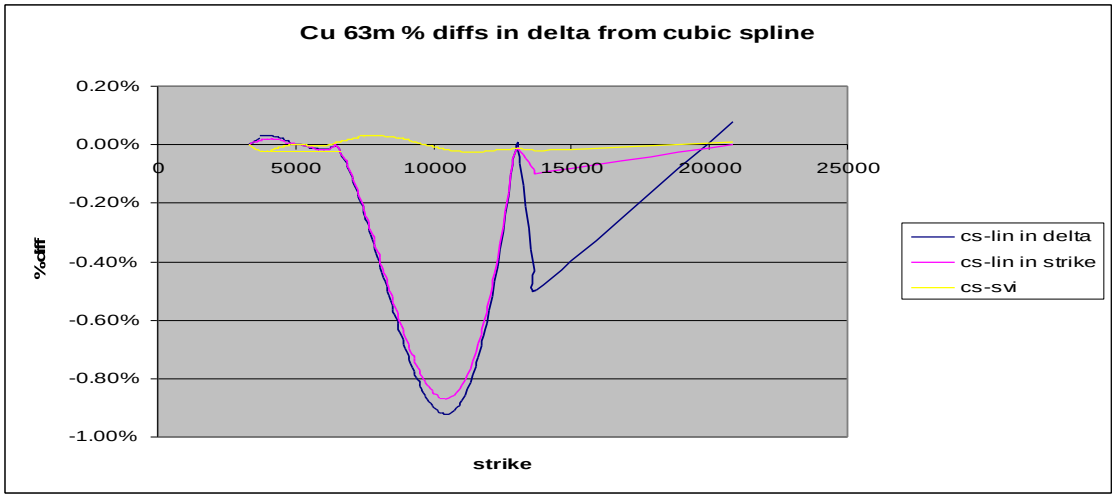

Zn 63m Actual diff in vol to cs
0.04
0.02
0.00
-0.02
-0.04
-0.06
-0.08
-0.10
-0.12
-0.14
-0.16
0
1000
2000
3000
4000
5000
6000
7000
8000
diff
strike
cs-SVI
cs-lin in strike
cs-lin in delta

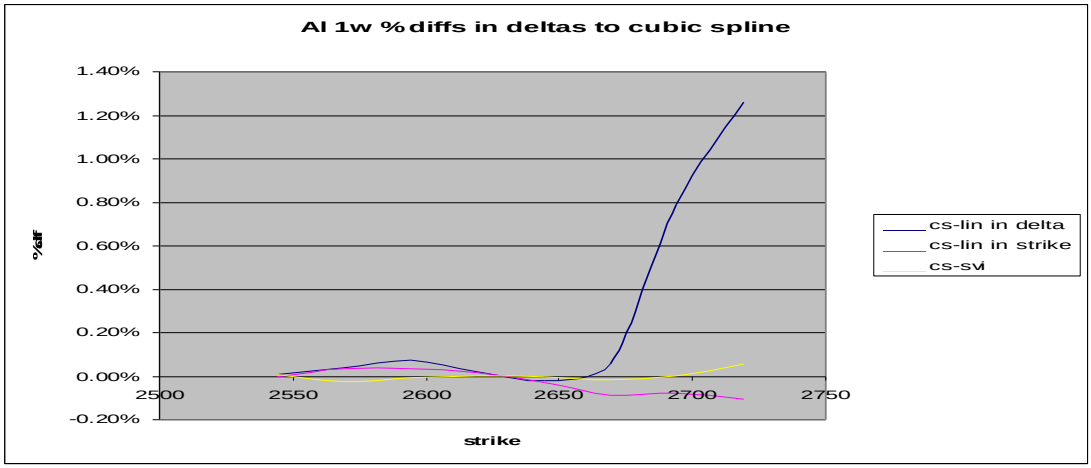
Appendix 3B - % Differences in Delta From Cubic Spline

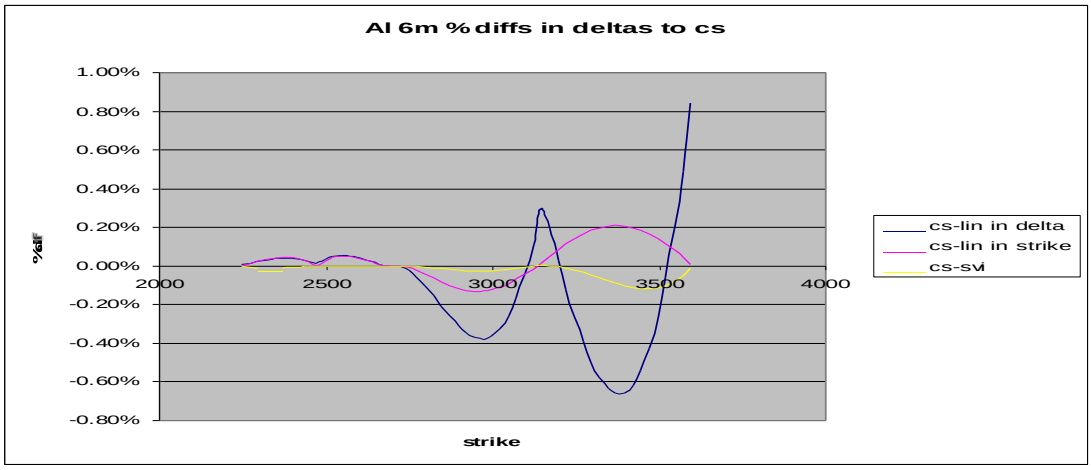


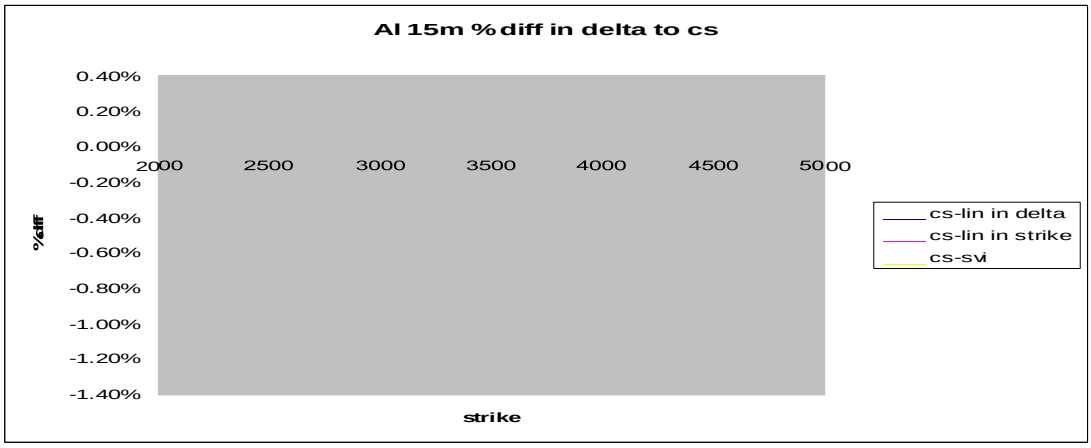


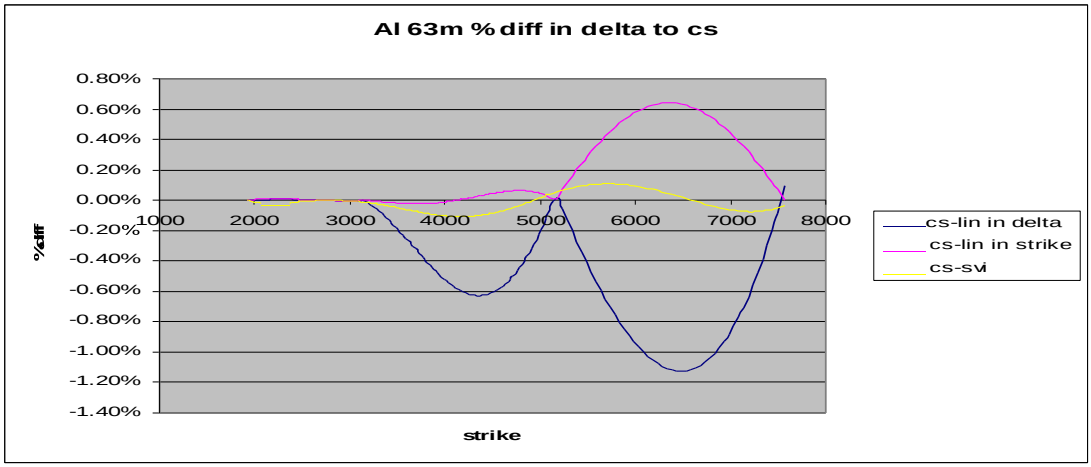


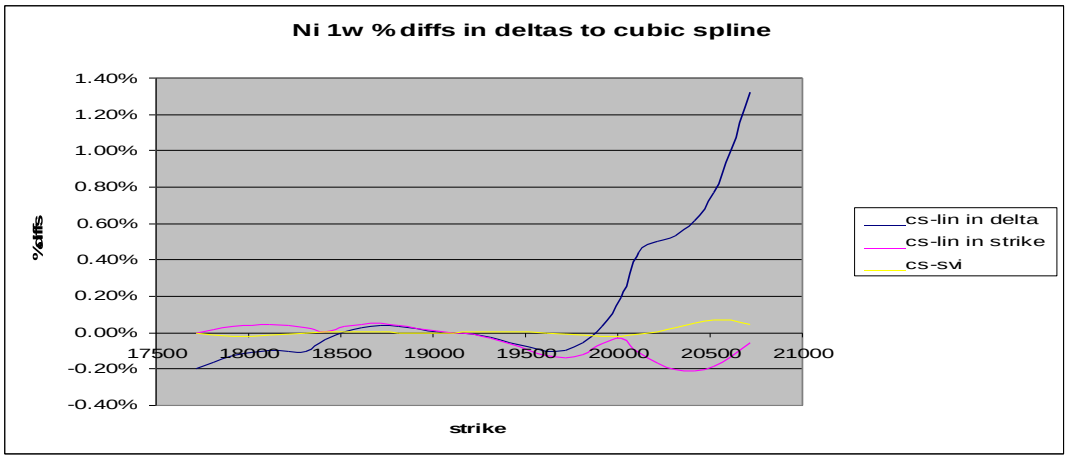


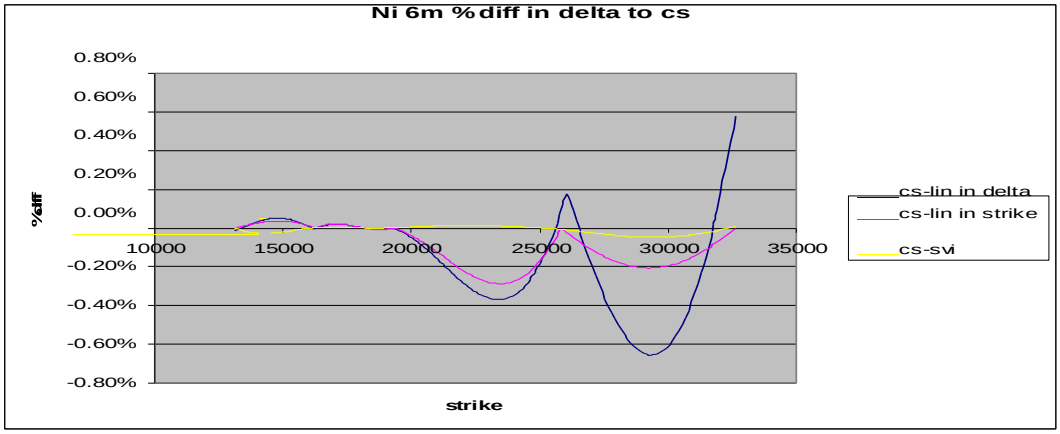

Al 1w % diffs in deltas to cubic spline
1.40%
1.20%
1.00%
0.80%
0.60%
0.40%
0.20%
0.00%
-0.20%
%diff
2500
2550
2600
2650
2700
2750
strike
cs-lin in delta
cs-lin in strike
cs-svi

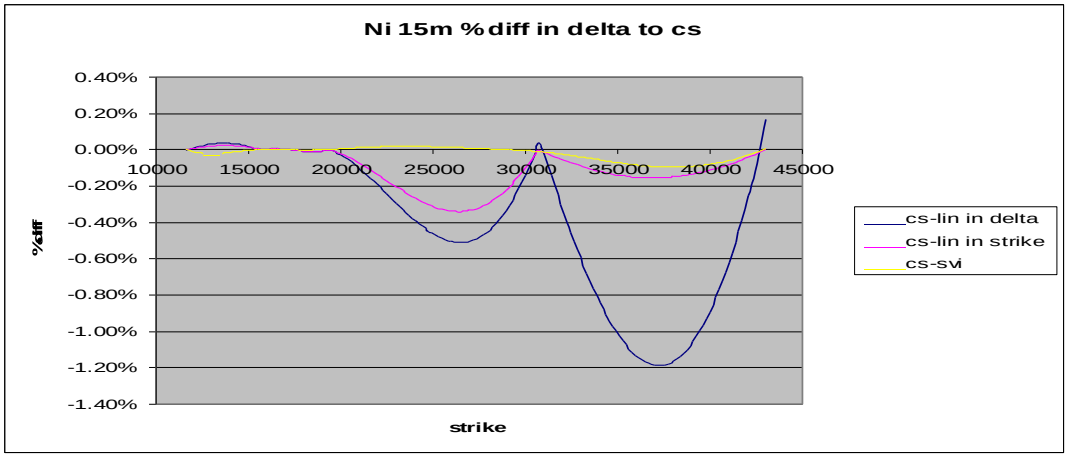

Al 6m % diffs in deltas to cs
1.00%
0.80%
0.60%
0.40%
0.20%
0.00%
-0.20%
-0.40%
-0.60%
-0.80%
%diff
2000
2500
3000
3500
4000
strike
cs-lin in delta
cs-lin in strike
cs-svi

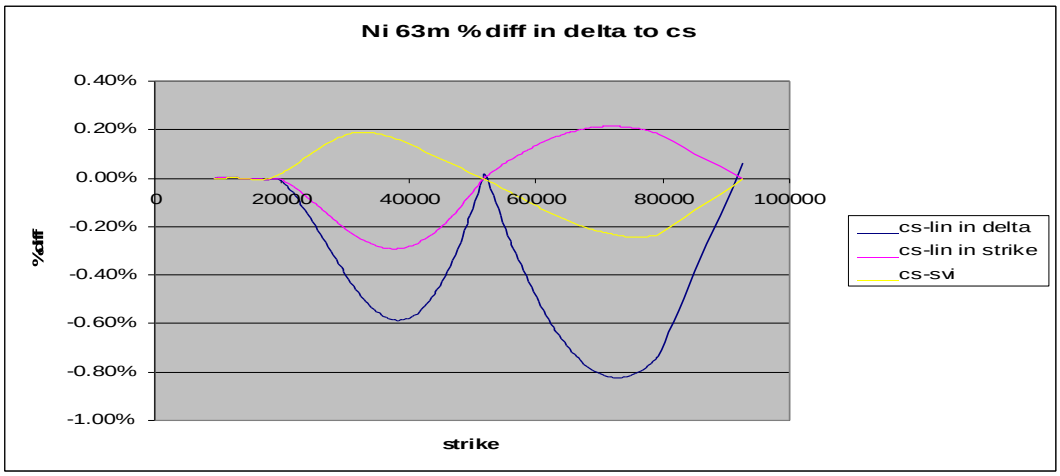

Al 15m % diff in delta to cs
0.40%
0.20%
0.00%
-0.20%
-0.40%
-0.60%
-0.80%
-1.00%
-1.20%
-1.40%
%diff
2000
2500
3000
3500
4000
4500
5000
strike
cs-lin in delta
cs-lin in strike
cs-svi

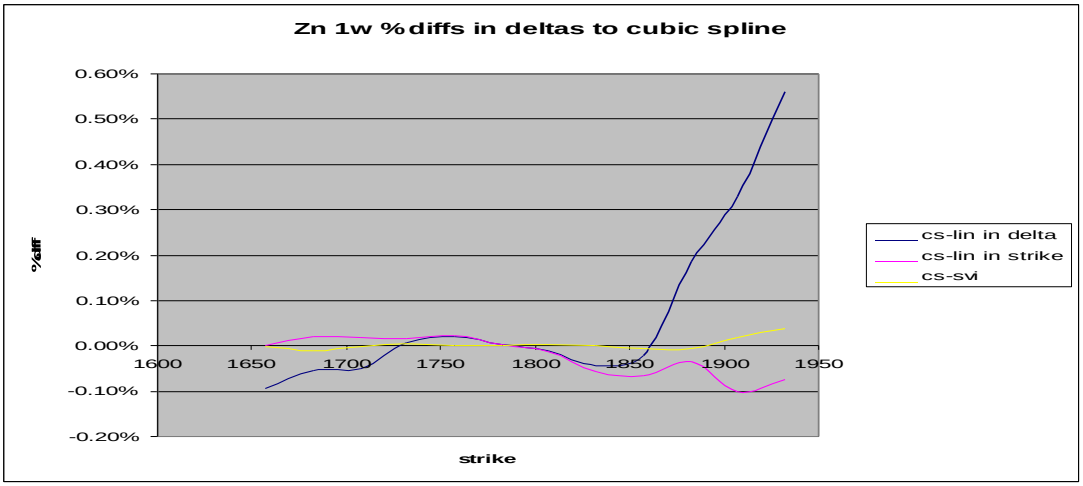

Al 63m % diff in delta to cs
0.80%
0.60%
0.40%
0.20%
0.00%
-0.20%
-0.40%
-0.60%
-0.80%
-1.00%
-1.20%
-1.40%
%diff
1000
2000
3000
4000
5000
6000
7000
8000
strike
cs-lin in delta
cs-lin in strike
cs-svi

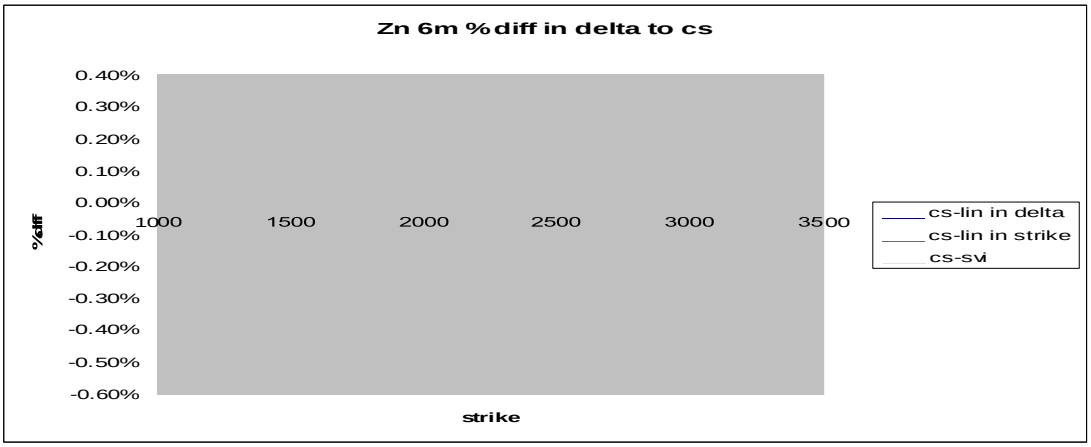

Ni 1w % diffs in deltas to cubic spline
1.40%
1.20%
1.00%
0.80%
0.60%
0.40%
0.20%
0.00%
-0.20%
-0.40%
%diffs
17500
18000
18500
19000
19500
20000
20500
21000
strike
cs-lin in delta
cs-lin in strike
cs-svi

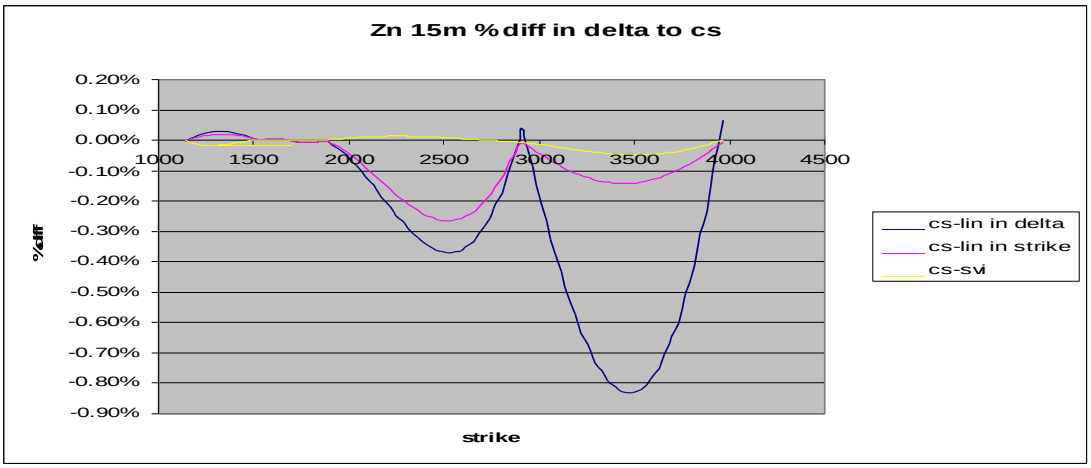

Ni 6m % diff in delta to cs
0.80%
0.60%
0.40%
0.20%
0.00%
-0.20%
-0.40%
-0.60%
-0.80%
%diff
10000
15000
20000
25000
30000
35000
strike
cs-lin in delta
cs-lin in strike
cs-svi

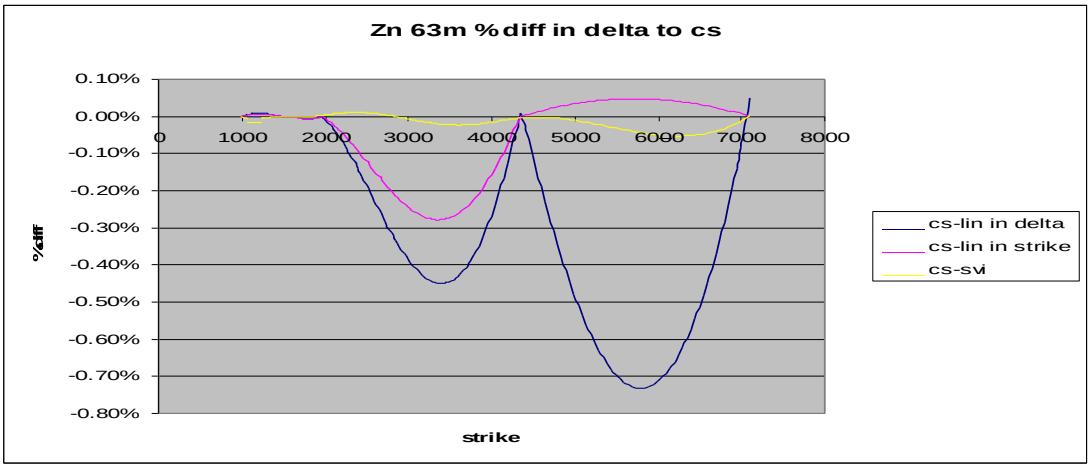

Ni 15m % diff in delta to cs
0.40%
0.20%
0.00%
-0.20%
-0.40%
-0.60%
-0.80%
-1.00%
-1.20%
-1.40%
%diff
10000
15000
20000
25000
30000
35000
40000
45000
strike
cs-lin in delta
cs-lin in strike
cs-svi


Ni 63m % diff in delta to cs
0.40%
0.20%
0.00%
-0.20%
-0.40%
-0.60%
-0.80%
-1.00%
%diff
0
20000
40000
60000
80000
100000
strike
cs-lin in delta
cs-lin in strike
cs-svi

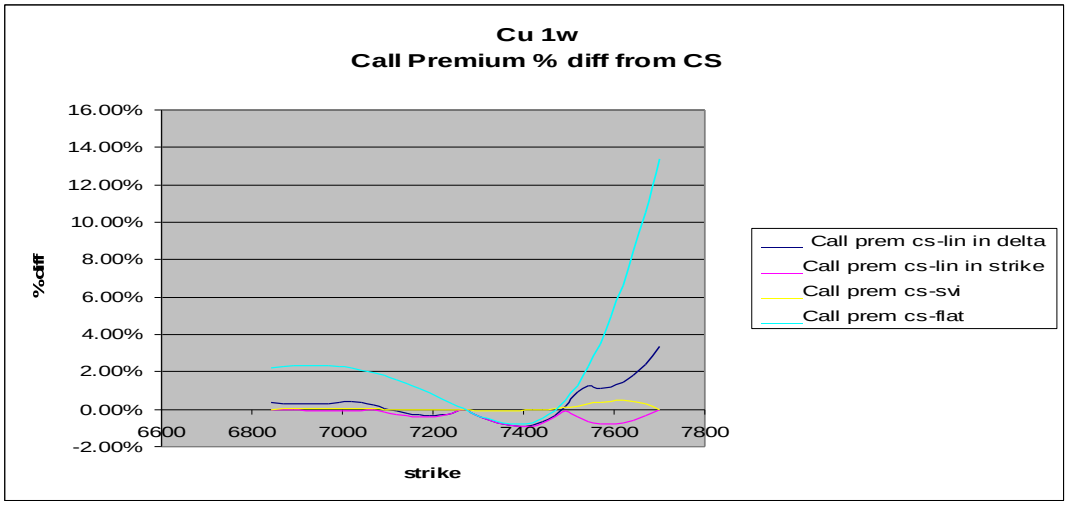

Zn 1w % diffs in deltas to cubic spline
0.60%
0.50%
0.40%
0.30%
0.20%
0.10%
0.00%
-0.10%
-0.20%
%diff
1600
1650
1700
1750
1800
1850
1900
1950
strike
cs-lin in delta
cs-lin in strike
cs-svi

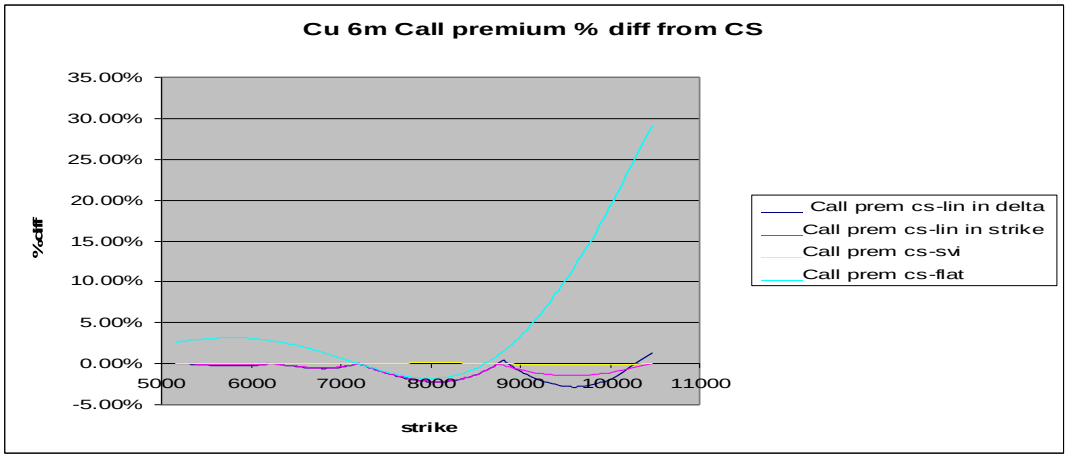

Zn 6m % diff in delta to cs
0.40%
0.30%
0.20%
0.10%
0.00%
-0.10%
-0.20%
-0.30%
-0.40%
-0.50%
-0.60%
%diff
1000
1500
2000
2500
3000
3500
strike
cs-lin in delta
cs-lin in strike
cs-svi

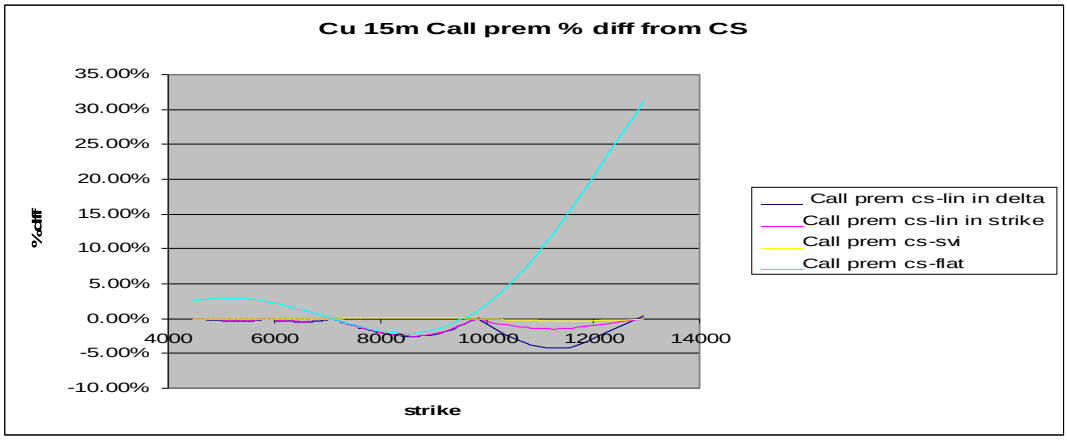

Zn 15m % diff in delta to cs
0.20%
0.10%
0.00%
-0.10%
-0.20%
-0.30%
-0.40%
-0.50%
-0.60%
-0.70%
-0.80%
-0.90%
%diff
1000
1500
2000
2500
3000
3500
4000
4500
strike
cs-lin in delta
cs-lin in strike
cs-svi

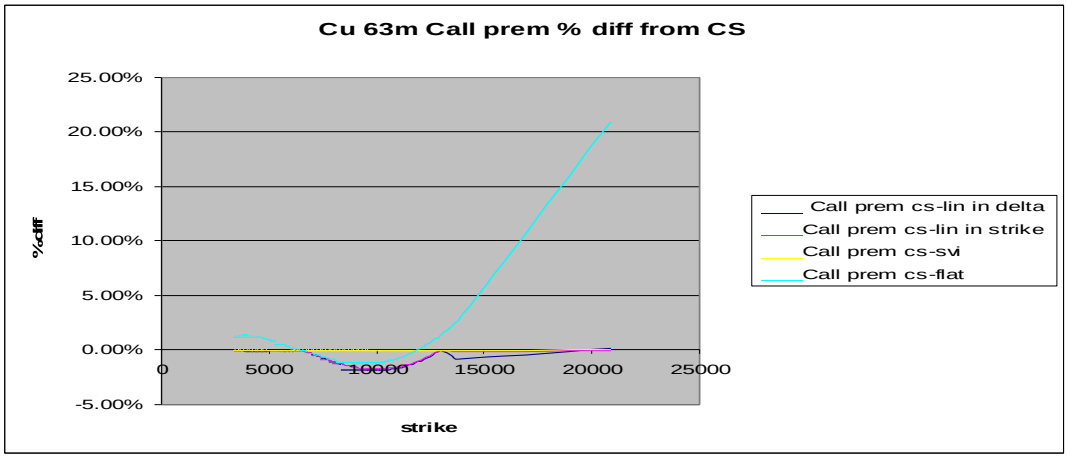

Zn 63m % diff in delta to cs
0.10%
0.00%
-0.10%
-0.20%
-0.30%
-0.40%
-0.50%
-0.60%
-0.70%
-0.80%
%diff
0
1000
2000
3000
4000
5000
6000
7000
8000
strike
cs-lin in delta
cs-lin in strike
cs-svi

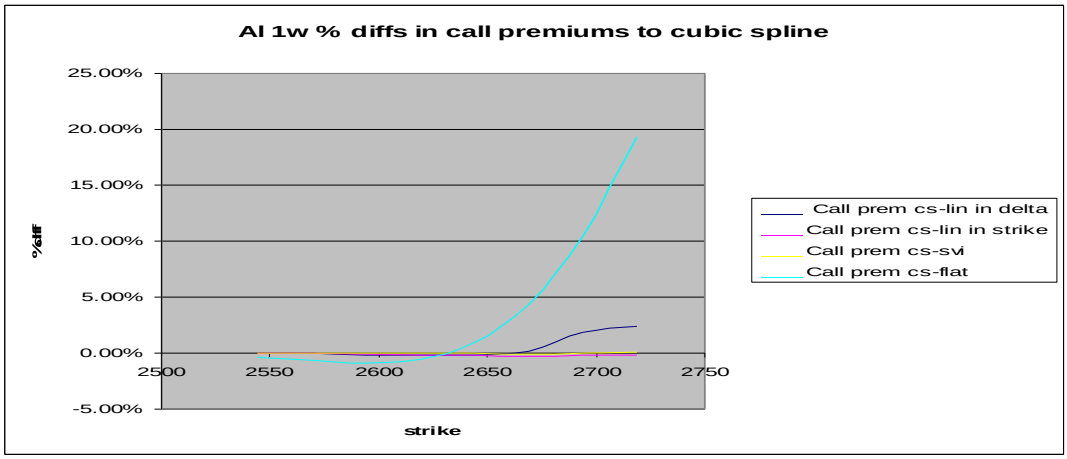
Appendix 3C - Call Premium % Differences from Cubic Spline

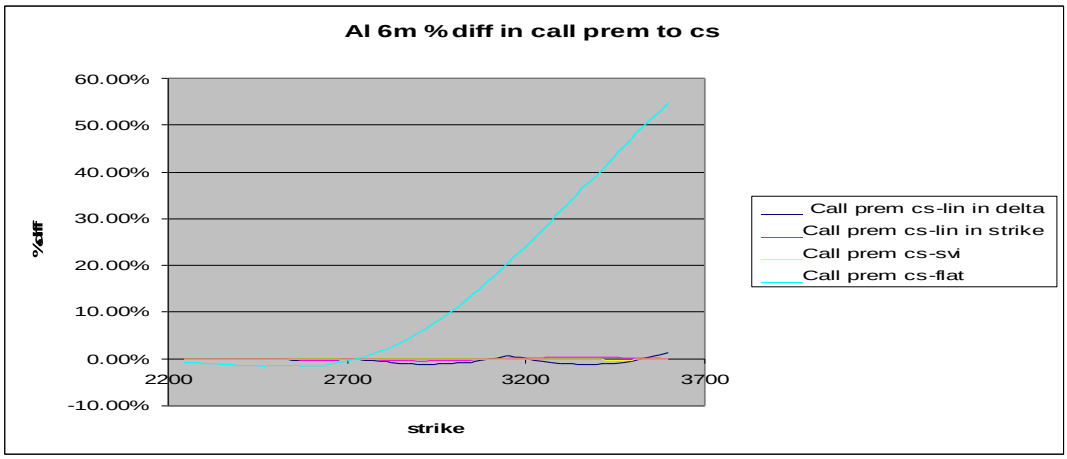


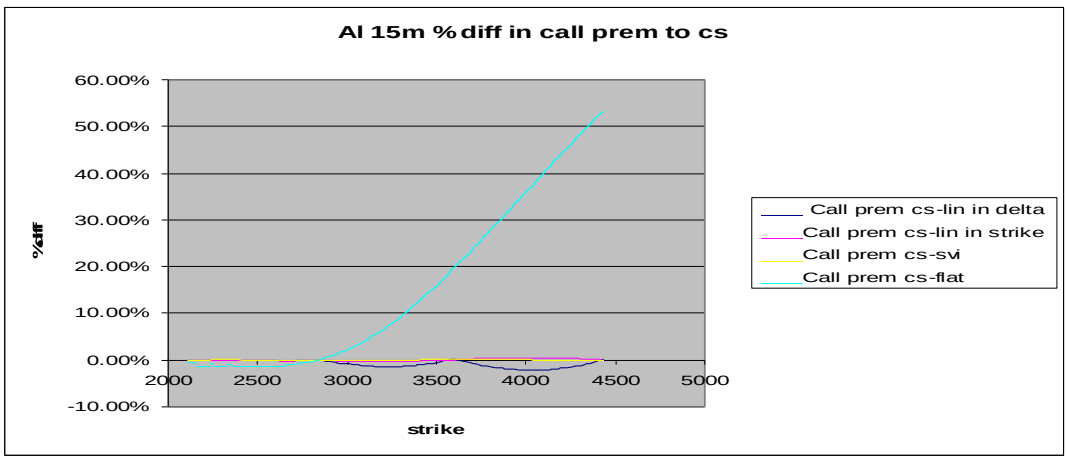


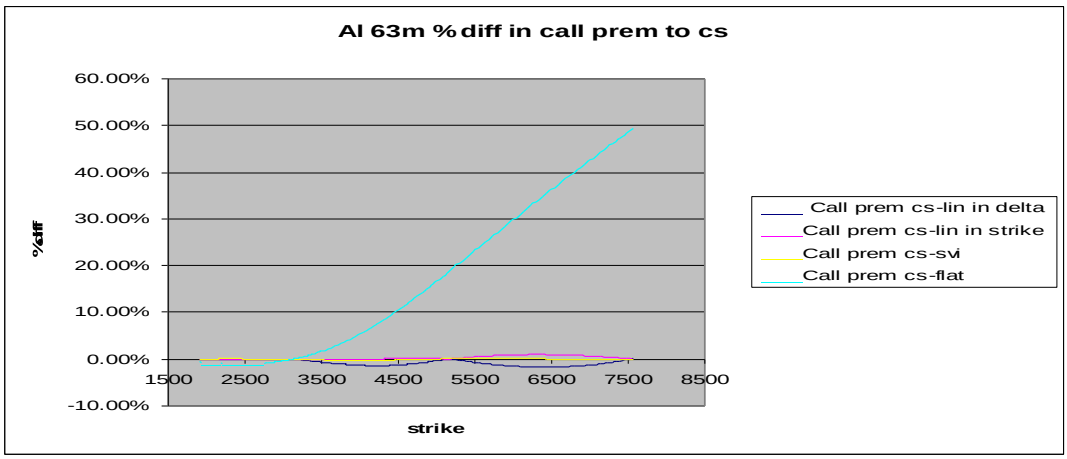


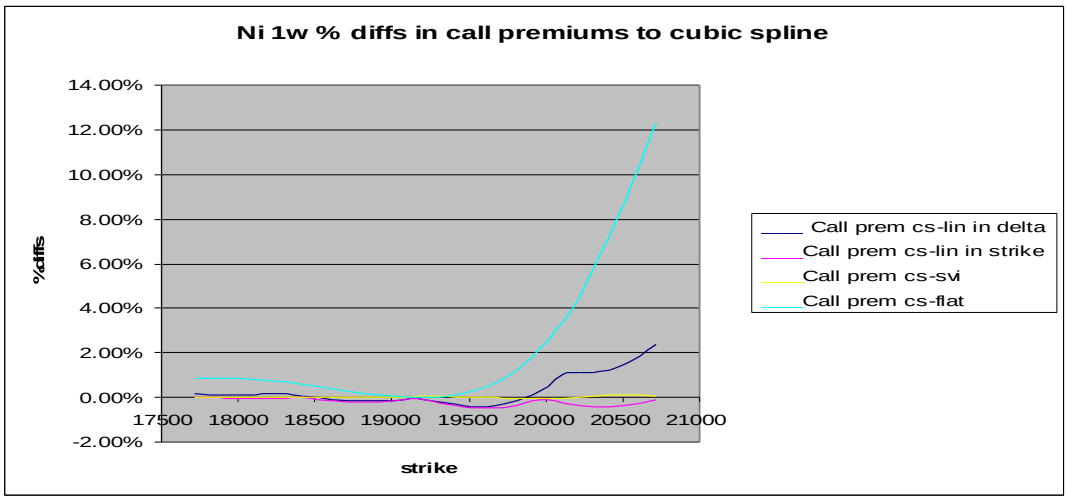


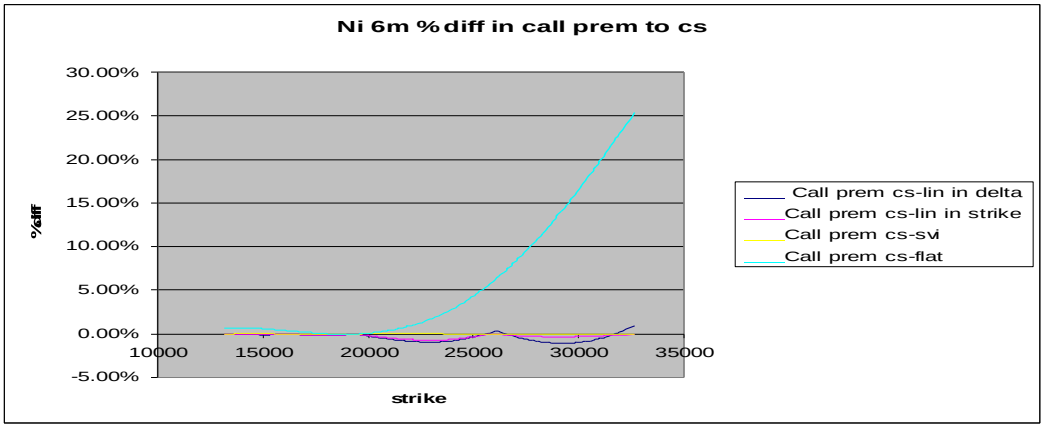

Al 1w % diffs in call premiums to cubic spline
25.00%
20.00%
15.00%
10.00%
5.00%
0.00%
-5.00%
%diff
2500
2550
2600
2650
2700
2750
strike
Call prem cs-lin in delta
Call prem cs-lin in strike
Call prem cs-svi
Call prem cs-flat

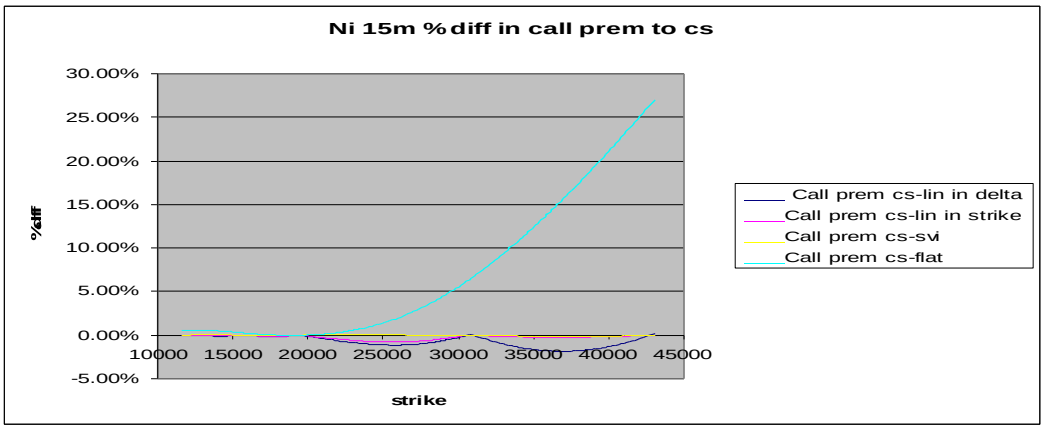

Al 6m % diff in call prem to cs
60.00%
50.00%
40.00%
30.00%
20.00%
10.00%
0.00%
-10.00%
%diff
2200
2700
3200
3700
strike
Call prem cs-lin in delta
Call prem cs-lin in strike
Call prem cs-svi
Call prem cs-flat

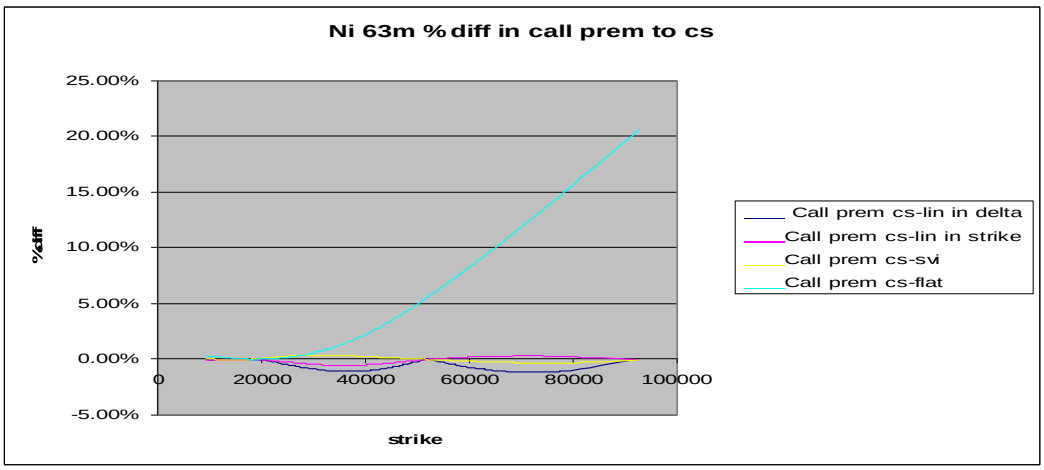

Al 15m % diff in call prem to cs
60.00%
50.00%
40.00%
30.00%
20.00%
10.00%
0.00%
-10.00%
%diff
2000
2500
3000
3500
4000
4500
5000
strike
Call prem cs-lin in delta
Call prem cs-lin in strike
Call prem cs-svi
Call prem cs-flat

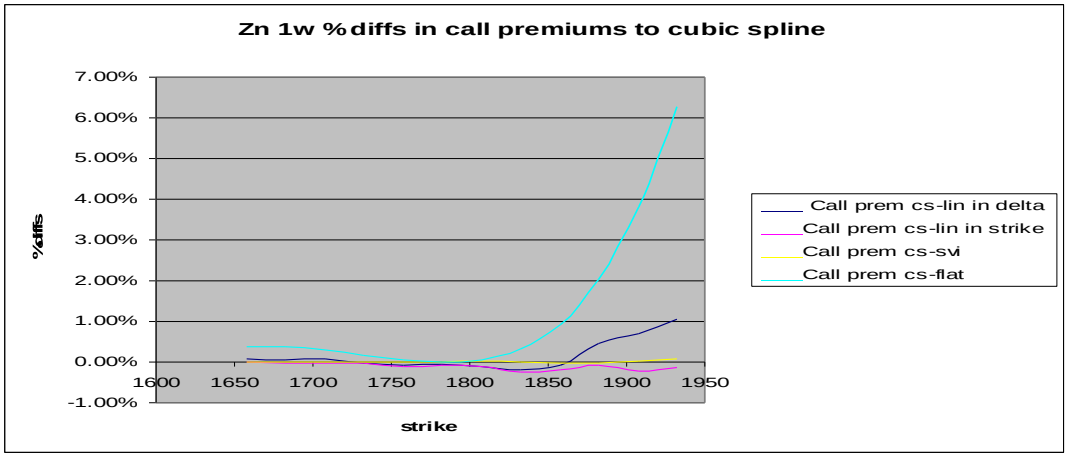

Al 63m % diff in call prem to cs
60.00%
50.00%
40.00%
30.00%
20.00%
10.00%
0.00%
-10.00%
%diff
1500
2500
3500
4500
5500
6500
7500
8500
strike
Call prem cs-lin in delta
Call prem cs-lin in strike
Call prem cs-svi
Call prem cs-flat

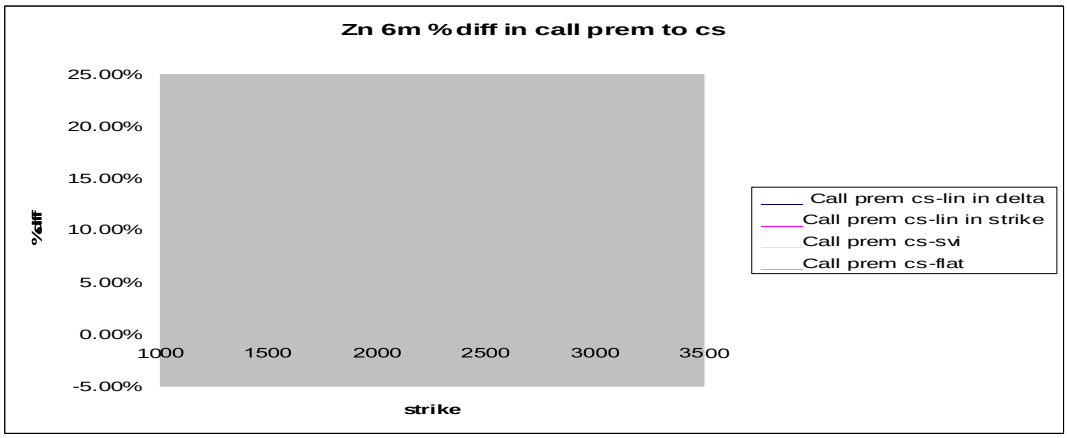

Ni 1w % diffs in call premiums to cubic spline
14.00%
12.00%
10.00%
8.00%
6.00%
4.00%
2.00%
0.00%
-2.00%
%diffs
17500 18000 18500 19000 19500 20000 20500 21000
strike
Call prem cs-lin in delta
Call prem cs-lin in strike
Call prem cs-svi
Call prem cs-flat

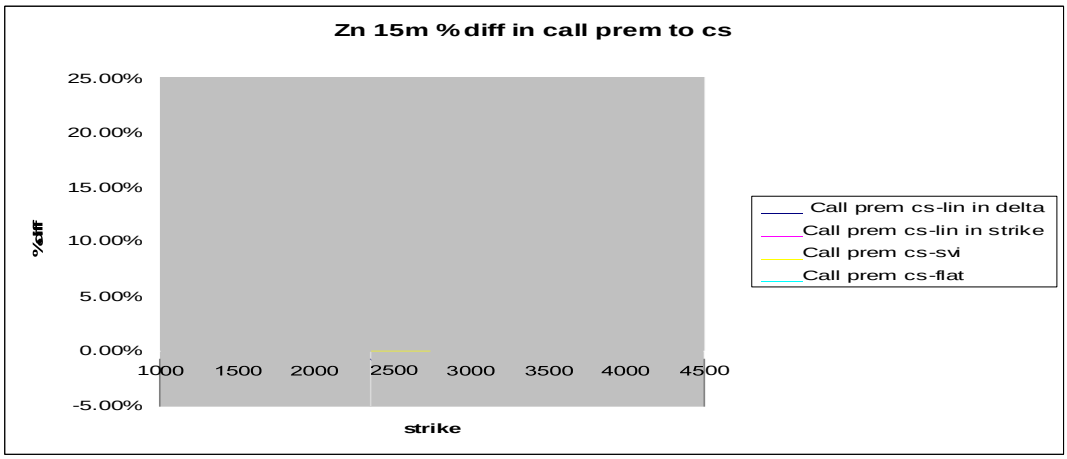

Ni 6m % diff in call prem to cs
30.00%
25.00%
20.00%
15.00%
10.00%
5.00%
0.00%
-5.00%
%diff
10000 15000 20000 25000 30000 35000
strike
Call prem cs-lin in delta
Call prem cs-lin in strike
Call prem cs-svi
Call prem cs-flat

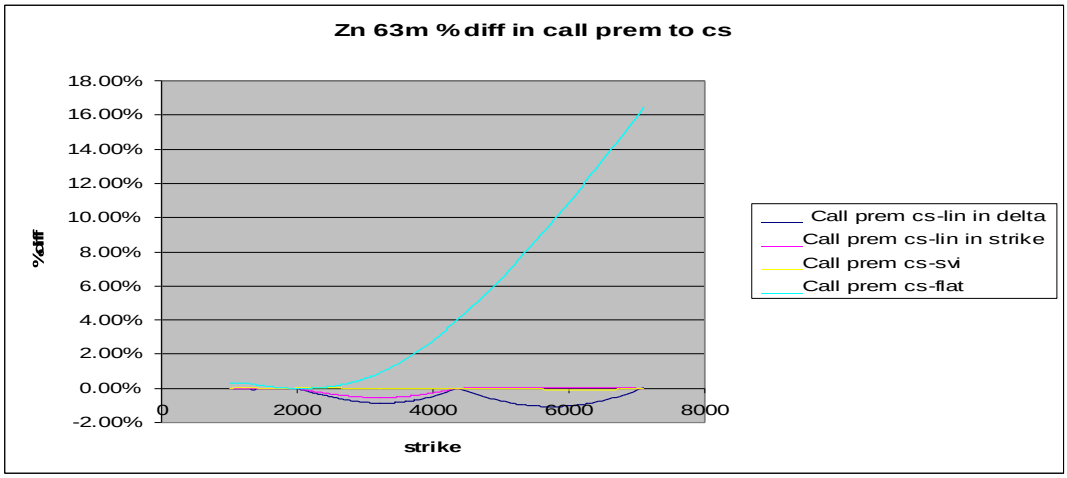

Ni 15m % diff in call prem to cs
30.00%
25.00%
20.00%
15.00%
10.00%
5.00%
0.00%
-5.00%
%diff
10000 15000 20000 25000 30000 35000 40000 45000
strike
Call prem cs-lin in delta
Call prem cs-lin in strike
Call prem cs-svi
Call prem cs-flat


Ni 63m % diff in call prem to cs
25.00%
20.00%
15.00%
10.00%
5.00%
0.00%
-5.00%
%diff
0 20000 40000 60000 80000 100000
strike
Call prem cs-lin in delta
Call prem cs-lin in strike
Call prem cs-svi
Call prem cs-flat


Zn 1w % diffs in call premiums to cubic spline
7.00%
6.00%
5.00%
4.00%
3.00%
2.00%
1.00%
0.00%
-1.00%
%diffs
1600
1650
1700
1750
1800
1850
1900
1950
strike
Call prem cs-lin in delta
Call prem cs-lin in strike
Call prem cs-svi
Call prem cs-flat


Zn 6m % diff in call prem to cs
25.00%
20.00%
15.00%
10.00%
5.00%
0.00%
-5.00%
%diff
1000
1500
2000
2500
3000
3500
strike
Call prem cs-lin in delta
Call prem cs-lin in strike
Call prem cs-svi
Call prem cs-flat


Zn 15m % diff in call prem to cs
25.00%
20.00%
15.00%
10.00%
5.00%
0.00%
-5.00%
%diff
1000
1500
2000
2500
3000
3500
4000
4500
strike
Call prem cs-lin in delta
Call prem cs-lin in strike
Call prem cs-svi
Call prem cs-flat


Zn 63m % diff in call prem to cs
18.00%
16.00%
14.00%
12.00%
10.00%
8.00%
6.00%
4.00%
2.00%
0.00%
-2.00%
%diff
0
2000
4000
6000
8000
strike
Call prem cs-lin in delta
Call prem cs-lin in strike
Call prem cs-svi
Call prem cs-flat

Appendix 4 - Out of Sample Test for 50 Delta

In the volatility graphs, the cubic spline and SVI methods loop lower than linear, which cuts off straight. In fact the linear results predict the contributed 50 delta volatility better than the non-linear methods, which go too low in having to “turn the corner” at their troughs.

Cu Out of sample 50 delta test

	50d actual volat	SVI volat	lin volat	SVI resids	lin resids	cubic spline
1w	36.18	36.18	36.18	0.01	0.00	36.16
1m	36.06	35.97	36.05	0.09	0.00	35.97
3m	35.98	35.81	35.98	0.18	0.00	35.81
6m	35.02	34.76	35.01	0.26	0.00	34.77
9m	34.52	34.22	34.51	0.30	0.00	34.23
12m	33.99	33.66	33.99	0.33	0.00	33.68
15m	33.51	33.16	33.50	0.35	0.01	33.18
21m	32.82	32.43	32.81	0.39	0.01	32.45
27m	32.28	31.88	32.27	0.41	0.01	31.89
39m	31.53	31.11	31.52	0.42	0.01	31.12
51m	30.55	30.20	30.54	0.35	0.01	30.22
63m	29.80	29.54	29.79	0.26	0.01	29.55
123m	25.76	25.58	25.73	0.18	0.02	25.57
			Total error	3.50	0.09	

AL Out-of-sample 50 delta test

	50d	nonlin	lin	nonlin resids	lin resids
1w	23.13	23.13	23.13	0.00	0.00
1m	23.28	23.27	23.27	0.01	0.01
3m	24.39	24.36	24.36	0.03	0.03
6m	24.25	24.17	24.20	0.08	0.05
9m	24.32	24.26	24.25	0.05	0.07
12m	24.20	24.11	24.12	0.08	0.07
15m	24.11	24.02	24.03	0.09	0.08
21m	23.96	23.87	23.87	0.09	0.09
27m	23.67	23.59	23.58	0.09	0.10
39m	23.22	23.12	23.11	0.10	0.11
51m	22.96	22.86	22.85	0.11	0.12
63m	21.89	21.79	21.77	0.10	0.12
123m	18.28	18.21	18.16	0.07	0.12
			Total error	0.89	0.96

Ni Out of sample 50delta test

	50 d	nonlin	lin	nonlin resids	lin resids
1w	50.80	50.76	50.80	0.05	0.00
1m	50.28	50.19	50.27	0.09	0.01
3m	50.14	49.98	50.11	0.16	0.03
6m	48.20	48.01	48.16	0.19	0.04
9m	46.22	46.01	46.16	0.20	0.05
12m	44.24	44.01	44.19	0.24	0.06
15m	43.51	43.29	43.45	0.22	0.06
21m	42.10	41.86	42.02	0.24	0.07
27m	41.29	41.06	41.22	0.23	0.08
39m	39.33	39.14	39.25	0.20	0.08
51m	38.40	38.19	38.31	0.20	0.09
63m	37.98	37.71	37.89	0.27	0.09
123m	37.74	37.52	37.66	0.23	0.09
			Total error	2.52	0.74

Zn Out of sample 50delta test

	50 d	nonlin	lin	nonlin resids	lin resids
1w	52.87	52.85	52.87	0.03	0.00
1m	51.99	51.93	51.98	0.06	0.01
3m	50.12	50.02	50.10	0.10	0.02
6m	48.17	48.03	48.14	0.14	0.03
9m	46.11	45.96	46.08	0.15	0.03
12m	44.03	43.85	44.00	0.18	0.03
15m	42.17	42.01	42.13	0.16	0.04
21m	40.68	40.50	40.64	0.18	0.05
27m	39.10	38.96	39.07	0.14	0.03
39m	36.28	36.10	36.24	0.18	0.04
51m	34.00	33.88	33.96	0.12	0.04
63m	32.66	32.53	32.62	0.13	0.04
123m	27.30	27.15	27.24	0.15	0.06
			Total error	1.71	0.42